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MAGNETIC DIPOLE RADIATION AND THE ATMOSPHERIC ABSORPTION BANDS OF OXYGEN

By J. H. VAN VLECK

ABSTRACT[1]

It is shown that the atmospheric bands of oxygen are best interpreted as magnetic dipole radiation rather than as ordinary dipole or quadrupole radiation. Then the upper state can be ${}^1\Sigma_g^+$, as demanded by configuration theory, and still one can apply Schlapp's intensity formulas, which agree with experiment. Also Childs and Mecke's estimates of absolute intensities favor the magnetic dipole hypothesis. Some general remarks are included concerning the rôle of magnetic dipole radiation in molecular spectra. Certain satellite bands in *OH* which contradict the usual selection rules are most likely due to this type of radiation. Intensity formulas are given for the ${}^3\Sigma-{}^1\Delta$ oxygen band at about 12,600 A, which also arises from the magnetic dipole terms.

The atmospheric absorption bands of oxgyen have for their lower state the normal level ${}^3\Sigma_g^-$ of the oxygen molecule. Mulliken[2] showed that the upper state is a ${}^1\Sigma$ level, but there has been considerable question as to whether the precise type is ${}^1\Sigma_g^+$ or ${}^1\Sigma_u^-$. On the basis of the configuration theory developed by Mulliken, Hund, and others,[3] or more generally, with any rational ordering of the energy-

[1] This paper was presented at the Ann Arbor meeting of the American Physical Society, June, 1934. For explanation of the notation, which is fairly standard for molecular spectra, see R. S. Mulliken, *Phys. Rev.* **36**, 611, 1930, and *Rev. of Mod. Phys.*, **3**, 91, 1931.

[2] *Phys. Rev.*, **32**, 880, 1928.

[3] R. S. Mulliken, *Rev. of Mod. Phys.*, **4**, 1, 1932, especially Fig. 43 (also earlier papers in the *Phys. Rev.*); J. E. Lennard-Jones, *Trans. of the Faraday Soc.*, **25**, 668, 1929 (the first paper on "Atomic Orbitals"); F. Hund, *Zs. f. Physik*, **51**, 759; **63**, 719, 1928–30; G. Herzberg, *Leipziger Vorträge*, 1931, p. 167, or *Zs. f. Physik*, **57**, 601, 1929.

levels formed by the union of two oxygen atoms, it is much more reasonable that the upper state be $^1\Sigma_g^+$ than $^1\Sigma_u^-$, for the following reasons: The upper state of the atmospheric bands is only 1.6 volts above the normal level of the oxygen molecule, and has nearly the same internuclear distance r. A $^1\Sigma_u^-$ level would presumably lead to a larger energy difference than 1.6 volts, and would involve a larger value of r than would the ground level $^3\Sigma_g^-$. It is true that $^1\Sigma_u^-$ and $^1\Sigma_g^+$ have the same energy when the internuclear distance r is infinite, since both states can be derived, like the ground level, from the union of two normal ^3P oxygen atoms. However, the energies are different in the other limit $r=0$, and hence at actual, intermediate distances. Namely, $^1\Sigma_u^-$ requires the electron configuration $1s^2 2s^2 2p\sigma^2 2p\pi^3 3s^2 3p\sigma^2 3d\pi^3$ for Mulliken's "united atom" ($r=0$); whereas $^1\Sigma_g^+$ can be ascribed to the same configuration $1s^2 2s^2 2p\sigma^2 2p\pi^4 3s^2 3p\sigma^2 3d\pi^2$ as the normal molecule. The former configuration differs from the latter by the substitution of the anti-bonding electron $3d\pi$ for the bonding one $2p\pi$. This substitution will increase both the energy and the internuclear separation. If it influenced the energy by only 1.6 volts, one would expect a large number of other states derived from $^3P+{}^3P$ to be only a volt or two above the ground level. Actually, only one deep state is found experimentally, other than the levels connected with the atmospheric bands. This is the state found by Ellis and Kneser, also by Herzberg,[4] at 0.98 volts, and, as they note, is nicely classified as $^1\Delta_g$ if one interprets the atmospheric bands as $^3\Sigma_g^- - {}^1\Sigma_g^+$. The three states which can arise from the configuration $1s^2 2s^2 2p\sigma^2 2p\pi^4 3s^2 3p\sigma^2 3d\pi^2$ are $^3\Sigma_g^-$, $^1\Sigma_g^+$, $^1\Delta_g$; and, furthermore, $^1\Delta_g$ should be approximately midway between the first two.[5] Hence the $^1\Sigma_g^+$ interpretation explains satisfactorily the number and relative positions of the deep states.

[4] J. W. Ellis and H. O. Kneser, *Phys. Rev.*, **44**, 420, 1933; G. Herzberg, *Nature*, **133**, 759, 1934. Ellis and Kneser observed the 0.98 volt or 12,600 A band in liquid oxygen. Herzberg was the first to detect it in the gaseous state in the laboratory, and analyze the rotational fine structure. This band is also noticeable in the solar absorption spectrum of Abbott and Freeman (private communication with Professor Mulliken; also cf. Ellis and Kneser, *Phys. Rev.*, **45**, 133, 1934). Indirect evidence for the existence of the $^1\Delta$ level was found previously by B. Lewis and G. von Elbe from specific heats (*ibid.*, **41**, 678, 1932), but they were thus led to locate this state somewhat too low, viz., at 0.75 ± 0.05, rather than 0.98, volts above the ground level.

[5] E. Hückel, *Zs. f. Physik*, **60**, 442, 1930; also Mulliken, *loc. cit.*

On the other hand, study of the rotational intensity structure has strongly favored a ${}^1\Sigma_u^-$ assignment to the upper state of the atmospheric bands. By the "rotational intensity structure" we mean such items as the relative intensities of the P, Q, R branches, and the dependence of the intensity upon the component of the triplet; in other words, how the intensity is related to the quantum numbers J and K. Because of the intersystem nature of the atmospheric bands, the intensity formulas of Hönl and others commonly given in the literature cannot be directly applied. Instead, it is necessary to make somewhat more elaborate calculations inclusive of spin-orbit perturbations. Such calculations have been made by R. Schlapp[6] on the basis of the ${}^1\Sigma_u^-$ assignment. The intensity formulas which he obtains are in good agreement with the experimental data of Childs and Mecke.[7]

The reason that Schlapp utilized the ${}^1\Sigma_u^-$ assignment, when configuration theory favored ${}^1\Sigma_g^+$, is that the ordinary dipole selection rules allow only u and g states to combine. Thus, according to these rules, transitions from a ${}^1\Sigma_g^+$ upper level to the ground state ${}^3\Sigma_g^-$ would be forbidden even when spin-orbit perturbations are included. On the ${}^1\Sigma_g^+$ assignment, the atmospheric bands have usually been interpreted as quadrupole lines, since the quadrupole rules allow u–u and g–g transitions. The situation regarding intensities is then very unsatisfactory. In the first place, intersystem quadrupole lines are very faint—in fact, so extremely weak as to be inadequate in absolute intensity even though the atmospheric bands are quite feeble. Furthermore, the formulas which Rubinowicz[8] has developed for quadrupole lines require the existence of transitions by two units in J, with a probability of the same order of magnitude as for zero and unit changes in J. Actually no transitions by more than one unit in J are found in the atmospheric bands.

It is the purpose of the present article to show that this dilemma is removed if the atmospheric bands are interpreted as magnetic dipole radiation. Then the upper state can be ${}^1\Sigma_g^+$, as is required by con-

[6] R. Schlapp, *Phys. Rev.*, **39**, 806, 1932.

[7] W. H. J. Childs and R. Mecke, *Zs. f. Physik*, **68**, 344, 1931.

[8] A. Rubinowicz, *ibid.*, **65**, 662, 1932; *Ergebnisse der exakten Naturwissenschaften*, **11**, 170, 1932.

figuration theory, and still the intensity relations are satisfactory. Usually one thinks of radiation as arising from a variable electrical moment of the atom or molecule. Such radiation we may term "electric" or "ordinary" dipole radiation. There is also a feeble radiation attendant to a variable magnetic moment, which may be called "magnetic dipole radiation." The possibility of such radiation appears to have been first mentioned in connection with modern quantum theory by Brinkman,[9] who proved, among other things, that it was a necessary consequence of Dirac's theory of the electron. Magnetic dipole terms were included along with quadrupole ones in Stevenson's[10] study of the intensities of nebular lines. He did not explicitly isolate the quadrupole and magnetic dipole contributions in tabulating his computed intensities. More recently, in this journal, Condon[11, 12] independently treated the nebular lines, and delineated very clearly the contribution of the magnetic dipoles, showing that it often overshadows the effect of the Rubinowicz quadrupole terms. It was the latter feature that gave us the idea of interpreting the atmospheric bands as magnetic dipole radiation. Since these bands arise from molecules rather than atoms, they naturally present a somewhat different type of problem than the examples studied by Stevenson and Condon.

With the hypothesis of magnetic dipole radiation, the formulas for the rotational intensity structure are exactly the same as those developed by Schlapp for ordinary electric dipole radiation, and hence are in agreement with experiment. Our ability thus to appropriate all the final results of Schlapp's paper to our magnetic case is a consequence of the fact that both electric and magnetic moments transform like vectors under rotation of axes. This is really what is meant by "dipole." Hence both types of moment involve a repre-

[9] H. C. Brinkman, dissertation, Utrecht, 1932; *Physica*, **1**, 97, 1933.

[10] A. F. Stevenson, *Proc. R. Soc.*, A, **137**, 298, 1932.

[11] E. U. Condon, *Ap. J.*, **79**, 217, 1934.

[12] Shortly after the appearance of Condon's paper, J. Blaton also published a calculation on the nebular lines, likewise stressing the rôle of the magnetic dipole terms (*Zs. f. Physik*, **89**, 155, 1934). Blaton shows that magnetic dipole lines should exhibit an unusual type of polarization in the Zeeman effect. Professor Mulliken suggests to the writer that it would therefore be particularly interesting to make Zeeman measurements on the molecular lines which we attribute to magnetic dipoles.

sentation of degree 3 of the rotation group, and fortunately there is only one representation of a given degree for this group,[13] so that matrix elements are uniquely determined except for a proportionality factor. The magnetic moment differs from the electric in being even, rather than odd, as regards reflection in the origin (i.e., the substitution $x \to -x$, $y \to -y$, $z \to -z$), since the former is an operator of components

$$q\left(y\frac{\partial}{\partial z}-z\frac{\partial}{\partial y}\right),\quad q\left(z\frac{\partial}{\partial x}-x\frac{\partial}{\partial z}\right),\quad q\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial y}\right)\qquad \left(q=-\frac{he}{4\pi imc}\right)$$

whereas the latter has components $-ex$, $-ey$, $-ez$. From this property, one easily shows that magnetic dipole radiation connects the ground state $^3\Sigma_g^-$ with $^1\Sigma_g^+$ but not with $^1\Sigma_u^-$.[14]

Magnetic dipole radiation resembles quadrupole radiation in that, with both, the expression for the intensity involves the velocity of light in the inverse fifth power, as compared with the inverse third for ordinary dipole radiation. It is for this reason that some authors do not distinguish very clearly between magnetic dipole and quadrupole radiation. When we use the expression "quadrupole radiation," we always use the term in the strict sense of the word, thus meaning the part of the electric radiation which transforms like an irreducible quadratic form (e.g., xy, yz, xz, x^2-y^2, $x^2+y^2-2z^2$) under a rotation of axes, and which thus corresponds to a representation of degree 5 of the rotation group. Intensity formulas have been developed for quadrupole lines for atomic multiplets by Rubinowicz. Of course our case is a molecular one; but there is, nevertheless, sufficient resemblance to Rubinowicz' examples to make it clear that double transitions in J should be a notable feature in the atmospheric bands if the quadrupole terms are dominant.

Obviously, quadrupole radiation must be present concurrently with magnetic dipole, as either type can join $^3\Sigma_g^-$ and $^1\Sigma_g^+$. However,

[13] E. Wigner, *Gruppentheorie*, p. 167.

[14] When magnetic is substituted for ordinary dipole radiation, the relation $l=n^*$ on p. 809 of Schlapp's paper becomes $l=-n^*$. Our conclusions are consequences of this modification and the fact that the combinations become $++$ or $--$ rather than $+-$. Do not confuse this $+-$ symmetry with the sign choice indicated by the superscript attached to Σ, which has a different meaning. A Σ^+ state, for instance, can be either $+$ or $-$; cf. Mulliken, *loc. cit.*

the former is almost certainly faint compared to the latter, so that the latter effectively governs the rotational intensity structure. Condon shows that in general the ratio Q/M of the intensity of quadrupole to magnetic dipole radiation is

$$\frac{Q}{M}=\tfrac{3}{40}x\sigma^2\ ,$$

where σ is the ratio of the frequency of the line to the Rydberg constant, and where x is a ratio of certain matrix elements which is not vastly different from unity unless there are special selection rules operative which exclude one particular type of radiation. In the case of the atmospheric bands, σ is 0.12, so that Q/M has the value 10^{-3} if $x=1$. Actually, x is probably somewhat less than unity if we can reason at all by analogy with the atomic cases for which Condon has made numerical estimates of intensities. For instance, he computes $x=0.3$ for the line ${}^3P_1-{}^1D$ of O III. We therefore conclude that the quadrupole radiation is only about one ten-thousandth to one thousandth as strong as the magnetic dipole radiation. This difference in magnitude appears sufficient to explain why transitions by two units in J have not yet been observed. (In this connection we may note that the quadrupole lines having $\Delta J=2$ are slightly weaker than those having $\Delta J=1$. This effect introduces a factor of 4 or so handicapping the detection of $\Delta J=2$.) It would be very interesting to make a special search for lines involving $\Delta J=2$ on plates of sufficient exposure or sensitivity to exhibit lines one ten-thousandth as intense as the lines of the atmospheric bands usually observed.

So far we have appealed to configuration theory to show that the magnetic dipole interpretation based on a ${}^1\Sigma_g^+$ upper state is superior to the hypothesis of ordinary dipole radiation with a ${}^1\Sigma_u^-$ state. It is, however, not really necessary to go outside the realm of intensities to establish this superiority. It is true that both interpretations give the same rotational intensity structure, but this agreement only implies that relative intensities are alike. The absolute intensities are very different, and favor magnetic dipoles. According to Childs and Mecke, the mean life of the upper state of the atmospheric bands is 7 sec. With magnetic dipoles, one estimates a mean life of the

order of 100 sec., and with ordinary dipoles 10^{-3} sec.[15] Quadrupoles give 10^5 sec. Thus the magnetic dipoles yield a mean life intermediate between the two other possibilities, and agree by far the best with experiment. The discrepancy by a factor of 10 or so, even with these dipoles, should not be taken too seriously, as the foregoing estimates are only very rough (obtained by taking $a=1$, $b=0.01$ in the analogue of Condon's formula 15).

It is not improbable that there are other molecular bands besides the atmospheric oxygen bands which are due to magnetic dipole radiation. Intersystem magnetic radiations such as the atmospheric bands might often escape detection, as a mean life of 7 sec., after all, represents extreme faintness under ordinary conditions of observation. However, magnetic dipole lines need not necessarily be intersystem combinations; instead, they can be of a type which arises even when spin-orbit coupling is omitted. Then they are 10^4 times stronger than our previous estimate, and correspond to a mean life about 10^{-2} sec. (This is for $\lambda=7600$ A; there is the usual ν^3 factor in adapting to other wave-lengths.) Here there is an important difference from the atomic case. In the latter, magnetic dipole radiation can exist only by virtue of spin-orbit interaction, since otherwise **L** and **S**, the orbital and spin angular momenta, are constants of the motion. Obviously, radiation is possible only when there is an oscillating, rather than a constant, moment. In molecules only the component of **L** or **S** parallel to the figure axis is constant before the introduction of the spin-orbit perturbation or rotational distortion. Hence, without these complications, magnetic radiation can arise from, and only from, the perpendicular component. One can then only have $\Delta\Lambda=\pm 1$ if the orbit radiates magnetically, and $\Delta\Sigma=\pm 1$ if the spin does. In either case $\Delta\Omega=\Delta(\Lambda+\Sigma)=\pm 1$. The selection rules are thus somewhat different than in ordinary dipole radiation, where the parallel component can radiate and where consequently it is possible to have $\Delta\Lambda=\Delta\Sigma=0$. Another circumstance favoring magnetic dipole radiation in molecules is the fact that oftentimes the electric dipole radiation connects molecular states having the same dissociation products (or, more generally, dissociation products in

[15] These estimates of mean lives are all 10^4 greater than they would be otherwise, because the atmospheric bands are intersystem combinations.

which any given atom has the same Laporte parity). This electric radiation is then forbidden in the case of infinitely separated atoms, and exists only in virtue of the overlapping or mutual distortion of the wave functions of the two atoms. In such cases the electric or ordinary dipole radiation should be somewhat less than the usual estimates, though perhaps often not much less, since the non-orthogonality or overlapping factor is sometimes computed to be as great as one-fourth in stable molecules. On the other hand, magnetic dipole radiation thrives on transitions between states having the same dissociation products, since with infinitely separated atoms the magnetic moment is diagonal in everything but the inner quantum number, and so gives rise to non-vanishing transition probabilities only between states belonging to the same atomic configurations.[16]

In unsymmetrical molecules, magnetic dipole radiation, if detectable, should be evidenced by the presence of satellite lines which terminate on the wrong λ-doublet component from the standpoint of the ordinary selection rules. Satellites of precisely this character are found in the $^2\Sigma - {}^2\Pi$ band of *OH*.[17] Theoretically the ratio of the intensity of the satellite to the corresponding main line should be the same for the *P*, *Q*, and *R* branches, and of about the order of 10^{-5}. Also, this ratio should be independent of *J*. The independence of *J* is confirmed experimentally for the *Q* branch of the *OH* bands Less adequate experimental data are available for the *P* and *R* branches, but Jack's measurements[17] do seem at least to indicate that the satellites are present also for these branches. Kronig[18] attributed the satellites in *OH* to the breaking-down of the selection rules by stray external fields, but the magnetic dipole explanation is

[16] For infinitely separated atoms, the non-vanishing elements of the magnetic moment matrix connect only states of the same **L**, **S** even when the atomic configurations are the same. However, the **L**, **S** coupling can be broken down fairly easily by the molecular forces, so that it does not seem necessary that the dissociation products have the same values of **L** and **S** in order for magnetic dipole combinations to be developed in molecular spectra. Thus, in the *OH* case which we quote, both the upper and lower levels dissociate into $2p^4(O) + 1s$ (*H*); but in one case, the state of the configuration $2p^4$ which is involved is of type ^1D, while for the other it is ^3P.

[17] Heurlinger, dissertation, Lund, 1918; Fortrat, *J. de Physique*, **5**, 20, 1924; G. H. Dieke, *Proc. Amsterdam Acad.*, **28**, 174, 1925; also especially W. Watson, *Nature*, January 30, 1926; D. Jack, *Proc. R. Soc.*, A, **120**, 228, 1928; R. S. Mulliken, *Phys. Rev.*, **32**, 408, 1928.

[18] R. de L. Kronig, *Zs. f. Physik*, **50**, 353, 1927.

probably more satisfactory since it is not clear whether actually the external fields are large enough to be an appreciable factor.

In symmetrical molecules, magnetic dipole radiation gives rise to new bands rather than to satellites, since u–u or g–g transitions cannot join the same electronic states as do the ordinary u–g or g–u combinations. The atmospheric oxygen band is, in fact, an example of such a newcomer.

In closing, it may be well to summarize the selection rules for magnetic dipole radiation in diatomic molecules. This radiation connects only $+$ and $+$ and $-$ with $-$ states, so that the λ-doubling rules are the reverse of the ordinary. In addition, in symmetrical molecules it connects only u with u and g with g, thus yielding new bands instead of satellites as in the symmetrical case. If spin-orbit and rotational distortions are neglected, one or the other, but not both, of the quantum numbers Λ and Σ must change by one unit. Except for the differences in the $+-$ and u–g rules, the rotational intensity structure is the same as for ordinary dipole radiation. Thus these structures for ${}^1\Sigma_g^+ - {}^3\Sigma_g^+$, ${}^1\Sigma_g^- - {}^3\Sigma_g^+$, ${}^1\Sigma_g - {}^3\Pi_g$ are, for example, respectively[19] the same as those which Schlapp calculates for ${}^1\Sigma_g^+ - {}^3\Sigma_u^-$, ${}^1\Sigma_g^+ - {}^3\Sigma_u^+$, ${}^1\Sigma_g - {}^3\Pi_u$. He does not give the expressions for ${}^3\Sigma - {}^1\Delta$. Since, as we have already noted, a band of this type is actually found in oxygen at about 12,600 A (.98 volt) we shall add the intensity formulas for this case. These formulas apply equally well to ordinary and magnetic dipole ${}^3\Sigma - {}^1\Delta$ transitions. The oxygen band at 12,600 A is to be attributed to magnetic radiation, like the atmospheric band, since it is of the g–g rather than the g–u type.

	${}^3\Sigma_{J-1}$	${}^3\Sigma_J$	${}^3\Sigma_{J+1}$
${}^1\Delta$ $J'=J-1$	$\frac{(J-2)(J-1)(J+1)}{J(2J+1)}$	$\frac{(J-2)(J-1)}{J}$	$\frac{(J-2)(J-1)}{(2J+1)}$
${}^1\Delta$ $J'=J$	$\frac{(J+2)(J-1)}{J}$	$\frac{(J+2)(J-1)(2J+1)}{J(J+1)}$	$\frac{(J+2)(J-1)}{(J+1)}$
${}^1\Delta$ $J'=J+1$	$\frac{(J+2)(J+3)}{(2J+1)}$	$\frac{(J+2)(J+3)}{(J+1)}$	$\frac{J(J+2)(J+3)}{(J+1)(2J+1)}$

[19] The intensity formulas are unaltered if Σ^+, Σ^- be respectively substituted for Σ^-, Σ^+ in both the initial and final states, or if u, g be substituted for g, u. For instance ${}^1\Sigma_g^+ - {}^3\Sigma_g^-$ has the same rotational structure as ${}^1\Sigma_u^- - {}^3\Sigma_u^+$. In unsymmetrical molecules, the subscripts u and g are, of course, to be dropped.

In this table, J and J' relate respectively to $^3\Sigma$ and $^1\Delta$, while the subscript attached to Σ gives the value of K. For the $^1\Delta$ state, $J'=K'$. The formulas in the table give the intensity except for a constant proportionality factor, and are derived by the same methods as those given in Schlapp's paper.[6] These results agree with Herzberg's observation[4] that transitions with $K'-K=\pm 2$ are about as intense as those with $K'-K=0$, ± 1. Indeed, for large J all the entries in one row of the table are the same except for a factor of 2. Except for this observation, Herzberg's paper, a preliminary one, does not give any details concerning his measurements on intensities in the rotational structure. He does, however, report that the absolute intensity of the 12,600 A band is much lower than that of the atmospheric band. This fact can be understood in the following way: the atmospheric band has a non-vanishing intensity either because (*a*) spin-orbit interaction causes $^3\Sigma_1^-$ to borrow some of the characteristics of a $^1\Pi$ state or because (*b*) $^1\Sigma^+$ similarly borrows some of the properties of $^3\Pi_0$. The band at 12,600 A arises either because of (*a*) or because (*c*) $^1\Delta$ partakes somewhat of a $^3\Pi_2$ nature. If (*a*) were more important than (*b*) or (*c*), both bands would be of approximately equal absolute intensity, unless the Franck-Condon interference effects are radically different, which is unlikely. The observed intensity situation means that (*b*) is a more important borrowing effect than (*a*) or (*c*), presumably because of a more favorable location of the perturbing term.

The writer is indebted to Professor R. S. Mulliken for many valuable suggestions. He also wishes to thank Dr. W. W. Hansen for interesting discussions on magnetic radiation.

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SPECTROGRAPHIC STUDIES IN THE RED*

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ABSTRACT

I. *The spectrum of the companion of the Andromeda nebula* (M 32) has been photographed *to about* λ *7600*. Attempts have been made to find the forbidden Ca^+ lines 4^2S-3^2D ($\lambda\lambda$ 7291, 7324) in emission, but the low-dispersion spectra obtained give no certain evidence for the presence of these lines. The D lines of sodium are considerably stronger in the spectrum of the nebula than in that of the sun, but otherwise the two spectra are very similar.

II. A number of *stars*, both giants and dwarfs, *with H and K bright*, have been photographed in a higher dispersion to the same limit, λ 7600. These spectra also show no trace of the forbidden Ca^+ lines, although this result might have been expected because of the bright-line spectrum.

III. *A number of representative* M*-type giants and dwarfs have been observed* for the purpose of trying to find new luminosity effects *in the orange and red*. Several interesting differences between the spectra of giants and dwarfs have been found in these regions. The spectra of the M dwarfs have in general a much smoother appearance than those of M giants, notably because of the presence of stronger titanium oxide bands in giant spectra. *Three bands of calcium hydride* (*CaH*)—corresponding to the band-heads $\lambda\lambda$ 6389, 6382, $\lambda\lambda$ 6921, 6903, and $\lambda\lambda$ 7035, 7028—have been found in the spectra of M-type dwarfs but are hardly visible in the spectra of giants. These bands seem to give a valuable new luminosity criterion for M stars. *Very good evidence has also been found for the presence in* M *dwarf spectra of bands of magnesium hydride* with heads at $\lambda\lambda$ 5211, 4845. These bands seem to contribute to the smooth appearance in the M dwarf spectra of the *TiO* bands with heads at $\lambda\lambda$ 5167, 4955, and 4762, and appear also to be responsible for a general depression of the radiation-curve in this region.

The Ca lines $\lambda\lambda$ *6162, 6122, and 6102 are remarkably strong in late-type dwarfs* but faint or invisible in the giants. This result is in agreement with that found by Miss Burwell for stars of types K and M.

I

During a stay of two and one-half months at the Mount Wilson Observatory I had the opportunity of making some studies in the orange and red regions of the spectrum. The present paper describes the observations and presents the results thus far obtained from a study of the spectrograms.

In a brief paper in 1929[1] I pointed out that the calcium substratum in our galaxy may give rise to an observable emission spectrum and that the forbidden transitions 4^2S-3^2D, corresponding to $\lambda\lambda$ 7291.40, 7323.89, may be present as faint emission lines in the spectrum of the night sky. Up to the present these lines have not been observed.

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 498.

[1] *Nature*, **124**, 179, 1929; *Upsala Medd.*, No. 44, 1929.

The intensity of these lines is of course dependent on the density of the calcium substratum. If the density in our own galaxy is not high enough to make the lines appear, it seemed that the conditions of visibility might be more favorable in extra-galactic systems. In the case of a nebula the intensity of the lines should also be favored by the fact that, to a certain extent, we see straight through the nebula and not merely along a part of the diameter as we do when observing the night sky.

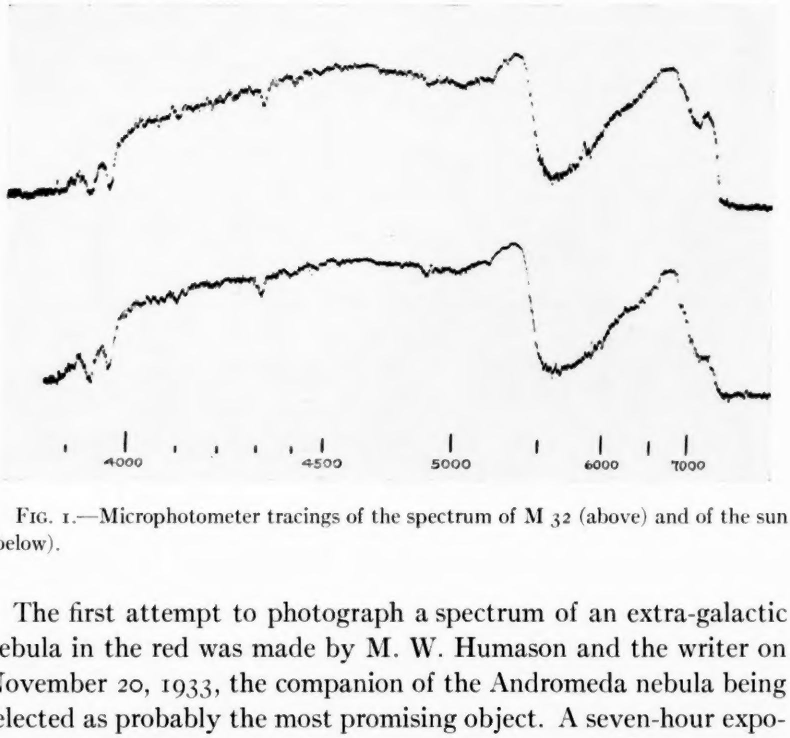

Fig. 1.—Microphotometer tracings of the spectrum of M 32 (above) and of the sun (below).

The first attempt to photograph a spectrum of an extra-galactic nebula in the red was made by M. W. Humason and the writer on November 20, 1933, the companion of the Andromeda nebula being selected as probably the most promising object. A seven-hour exposure with the 100-inch telescope and spectrograph No. 6 gave a somewhat overexposed spectrum to about λ 7600. The plate used was an Eastman I-N plate, bathed in ammonia.

A microphotometer tracing of this spectrum is shown in the upper part of Figure 1. The scale is about 8 times that of the original spectrum, which has a dispersion of 210 A/mm at λ 4500 and about 1000 A/mm at λ 7000.

The results do not give any clear evidence of the presence of the forbidden Ca^+ lines. The first somewhat overexposed image shows some faint irregularities at the wave-length where the lines should be, but a shorter exposure made later gives hardly any evidence for the presence of the lines; the dispersion is of course extremely small, and only strong lines could be detected from such spectra. It may be possible, however, to record the red spectral region of the same object with a considerably higher dispersion. During my short stay at Mount Wilson there was no time for such a trial.

Investigations of the red spectral region of extra-galactic nebulae are of great interest for our general knowledge of the proportions of stars of different spectral types in such systems. The spectrum we obtained is mainly that of the very concentrated nucleus of M 32. If this nucleus is composed of different spectral types, a spectrographic study of different spectral regions might give a method for at least a partial separation of the types. The spectral features of the red stars should be pronounced in the red part of the spectrum, whereas the early-type characteristics should be apparent in the ultra-violet.

In this connection a comparison between the spectrum of M 32 and that of the sun is of great interest. The lower tracing in Figure 1 is made from a spectrum of the sun (reflected sunlight and daylight) taken with the same spectrograph. The great similarity between the two tracings is very striking. The only apparent difference in spectral-line intensities is the remarkable strength of the D lines in the spectrum of M 32.[2] This may indicate a later spectral type in the yellow than in the blue for M 32, but we must also consider the possibility of interstellar absorption in our galaxy and within M 32, and also a possible intergalactic absorption. The somewhat redder color of M 32 may be due partly to an influence of Rayleigh scattering on the intensity distribution in the daylight spectrum.

The tracings give a nice illustration of the dwarf character of the spectra. The cyanogen band between λ 4144 and λ 4184 which, according to Lindblad,[3] is a very typical giant characteristic, is not visible in the spectrum of M 32.

[2] There is an indication also of a slightly stronger λ 4227 in the spectrum of M 32.

[3] *Mt. W. Contr.*, No. 228; *Ap. J.*, **55**, 85, 1922. *Upsala Medd.*, No. 28, 1927.

II

In connection with the search for the forbidden red Ca^+ doublet in the spectrum of M 32, it was of considerable interest to try to find this doublet in celestial spectra in general, as it has never been observed in the laboratory. A likely place seemed to be in the spectra of stars that show H and K as strong emission lines.[4] Our knowledge as to the origin of bright lines in stellar spectra is still very incomplete, but in some cases at least we have evidence for the opinion that the bright lines arise in a gaseous shell surrounding the star. In such a case the conditions may be more or less similar to those characterizing the planetary nebulae, and we may perhaps expect forbidden transitions to appear.

A list of late-type stars showing H and K bright has been given by Adams and Joy.[5] Most of the giants in this list show the emission lines double, of the type H_2, K_2. The dwarfs, on the other hand, show single lines. Two dwarfs which are very interesting in this connection are the companion of Mira Ceti[6] and Castor C.[7] H and K emission lines have also been observed in some variable stars; notably, stars of R Coronae Borealis type, which sometimes show them very strong.[8]

The new very sensitive Eastman I-N plates now make it possible to observe the spectra of dwarfs, as well as of giants, in the red with fairly good dispersion. The 60-inch telescope in conjunction with the ordinary Cassegrain spectrograph and 18-inch camera was used for this purpose. The dispersion is about 180 A/mm at λ 7000.

The following stars with bright H and K have been observed: α Ceti, η Geminorum (double H and K lines), and λ Andromedae, σ Geminorum, ϵ Eridani, 61^2 Cygni, and Castor C (single H and K lines). An underexposed spectrum of T Tauri (December 24, 1933), showing only $H\alpha$ and $H\beta$, was also obtained.

Although the spectra of these stars have been examined very carefully in the region around λ 7300, the forbidden Ca^+ doublet has not

4 As to the possible appearance of these lines in the flash spectrum, see Bowen and Menzel, *Pub. A.S.P.*, **40**, 332, 1928.

5 *Ibid.*, **43**, 407, 1931.

6 *Mt. W. Contr.*, No. 311; *Ap. J.*, **63**, 281, 1926.

7 *Mt. W. Contr.*, No. 320; *Ap. J.*, **64**, 250, 1926.

8 *Pub. A.S.P.*, **32**, 59, 1920; **35**, 325, 1923; **44**, 385, 1932.

been found. The lines cannot therefore have been present as strong emission lines. In order to detect faint lines superposed on the crenelated continuous background of late-type spectra, a still higher dispersion is needed.

III

From a preliminary inspection of the plates taken in connection with the search for the forbidden Ca^+ doublet, it was found that the

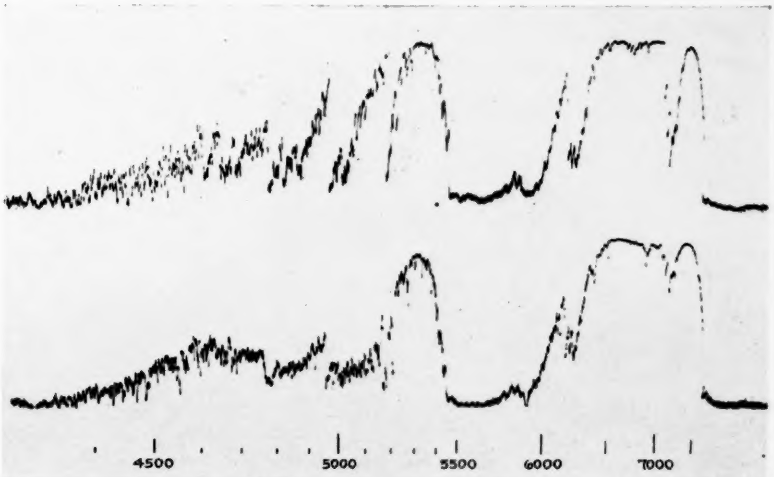

FIG. 2.—Microphotometer tracings of M-type spectra, giant (above) and dwarf (below). The stars are ρ Persei (M4, $M=-0.7$) and Boss 2935 (M2, $M=+10.4$).

giants and dwarfs of spectral type M show some noticeable differences in the red and yellow regions. For the purpose of a further study of these effects a selected number of late-type giants and dwarfs was observed with the 60-inch telescope, as follows: *dwarfs:* ϵ Eridani, 61^2 Cygni, Castor C, Groom. 34, Boss 2935; *giants:* α Arietis, α Tauri, α Ceti, β Andromedae, α Orionis, η and μ Geminorum, δ Virginis, and ρ Persei.

Figure 2 gives a good illustration of the difference in character of giant and dwarf spectra. The upper tracing is from a spectrogram of ρ Persei (M4, $M=-0.7$), the lower from Boss 2935 (M2, $M=+10.4$).[9] The most striking features of these tracings are the strong

[9] Spectral types and absolute magnitudes according to *Mt. W. Contr.*, No. 319; *Ap. J.*, **64**, 225, 1926.

well-known TiO bands,[10] and notably two bands in the red (heads at λλ 7054, 6159), and three in the green and blue (heads at λλ 5167, 4955, 4762). The band at λ 7125 is somewhat strengthened by the atmospheric band at λ 7200. The depression of the curves around λ 5600 is due to the low sensitivity of the plate in this region.

The Mount Wilson spectral classification of M stars is based mainly on the strength of the TiO bands. It is therefore natural that Boss 2935, type M2, should show fainter bands than ρ Persei of type M4. From the tracings it is apparent, however, that the intensity distribution within the green and blue bands is not the same for a giant and a dwarf, a difference shown by all the spectra studied. In the

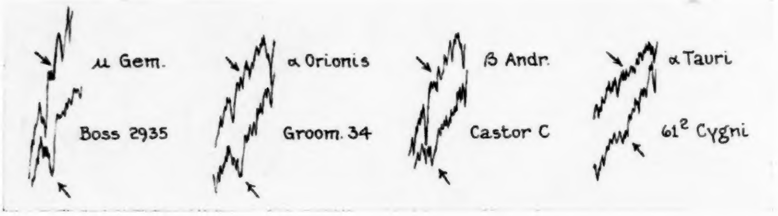

FIG. 3.—Tracings showing the λλ 6389, 6382 band of calcium hydride, present in M dwarf spectra but very faint or invisible in giant spectra. The band is marked with arrows. The strong line to the left of the band in the giants is the low-temperature line of iron λ 6358. The spectral types and absolute magnitudes are: α Tau, K5, +1.0; β And, Mo, +0.3; α Ori, M2, −4.3; μ Gem, M3, −0.4; and 61² Cyg, Mo, +8.4; Castor C, M2, +9.6; Groom 34, M2, +10.4; Boss 2935, M2, +10.4.

case of the giants the bands are very deep and the continuous spectrum on the violet side of the heads rises to high peaks. In the case of the dwarfs the bands are more or less smoothed out, and the spectrum has not the same crenelated appearance. As will be shown later, this difference between giants and dwarfs seems to be partly caused by the presence of magnesium hydride bands in the dwarf spectra.

According to recent theoretical investigations by Y. Cambresier and L. Rosenfeld[11] and by Russell,[12] the TiO bands should appear

[10] A. Fowler, *Proc. R. Soc. Lon.*, A, **79**, 509, 1907; F. Lowater, *Proc. Phys. Soc. Lon.*, **41**, 557, 1929; A. Christy, *Ap. J.*, **70**, 1, 1929.

[11] *M. N.*, **93**, 710, 1933.

[12] *Mt. W. Contr.*, No. 490; *Ap. J.*, **79**, 317, 1934.

stronger in M giants than in M dwarfs of the same temperature. This explains why very late M-type dwarfs (classified according to the strength of the *TiO* bands) have never been observed. It also explains why the M-type dwarfs that I observed show in general very faint bands as compared with the giants. The use of spectrophotometric methods for the study of these effects in the red (notably the λ 6159 band) seems to be a very promising method for the determination of stellar luminosities.[13] Because of the lack of intensity scales, the spectra studied here cannot be used for such photometric work.

From an inspection of the spectra it was found that a diffuse broad line at λ 6385 is present in the M dwarfs but hardly visible in the

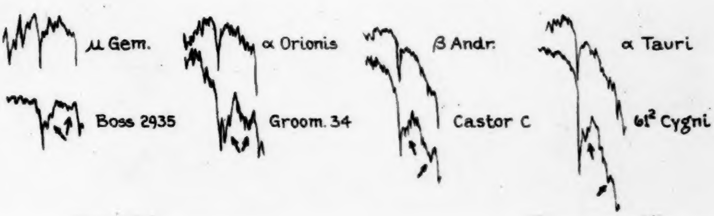

FIG. 4.—Tracings showing the λλ 6921, 6903 and λλ 7035, 7028 bands of calcium hydride in M dwarf spectra. The first band is somewhat overlapped by the atmospheric B band, seen in the tracings as a deep line to the left of the *CaH* band. For the spectral types, cf. Fig. 3.

giants. An attempt to identify this line with an atomic line did not give any positive result, but the wave-length was found to correspond exactly to that of a band of *CaH* with heads at λλ 6389, 6382.[14] The tracings in Figure 3 show the band very clearly in all the dwarfs examined, whereas the band is very weak or absent in the tracings of the giants.

This band is the strongest in the spectrum of calcium hydride. Two other bands are situated at λλ 6921, 6903 and at λλ 7035, 7028. As seen in Figure 4, these bands are also well visible in the tracings of the M dwarfs but are very weak or invisible in the giants. The identification of *CaH* in the M dwarf spectra seems therefore per-

[13] Cf. Lindblad's measurement of the cyanogen bands (*Mt. W. Contr.*, No. 228, p. 5; *Ap. J.*, **55**, 85, 1922).

[14] See Hulthén, *Phys. Rev.*, **29**, 97, 1927.

fectly certain because of the simultaneous appearance of these three bands.

The bands of *CaH* are well-known characteristics of the spectra of sun-spots,[15] but they have never before been observed in stellar spectra. With the dispersion used the bands have the appearance of diffuse lines. The λλ 6389, 6382 band is very conspicuous. These *CaH* bands seem to give us a new and promising luminosity criterion for M-type stars. In the dwarf spectra the bands first appear at about Mo and increase in intensity with advancing spectral type. In the giant spectra the bands are faint in all subdivisions of M observed by me.

The strong intensity of the *CaH* bands in dwarf M spectra relative to those in giant M spectra can be explained partly by the smaller degree of ionization of *Ca* in the case of the dwarfs. According to a private communication from Dr. Russell, however, most of the effect seems to be involved in the relationship between the general absorption coefficient and the pressure.

As bands of *CaH* have been found in dwarf M spectra, we have reason to believe that bands of *MgH* are also present. Bands of *MgH* are fairly strong in the spectra of sun-spots,[16] and they also seem to appear sometimes in the spectrum of Mira Ceti.[17]

Figure 5 shows a microphotometric tracing of the Mo dwarf 61^2 Cygni. This star, as well as the other M dwarfs observed, shows the *Mg* triplet (λλ 5183, 5172, and 5167) much stronger and broader than the M giants. At λ 5211 there is in the dwarf spectra a very strong and broad line which is not so conspicuous in the giants. All the dwarfs show a very apparent band at about λ 4788 which is not well seen in the giants, but the *TiO* band at λ 4762 partly overlaps it. In general, the continuous spectrum is somewhat depressed in the dwarfs compared with the giants in the region from λ 5211 to about λ 4700 (Fig. 2). To some extent, however, the great depression shown by Figure 5 in this region is due to the sensitivity-curve of the plate, as may be seen from the spectrum of the sun (Fig. 1).

[15] C. M. Olmsted, *Mt. W. Contr.*, No. 21; *Ap. J.*, **27**, 66, 1908; R. S. Richardson, *Mt. W. Contr.*, No. 422; *Ap. J.*, **73**, 216, 1931.

[16] Fowler, *Phil. Trans. R. Soc. Lon.*, A, **209**, 447, 1909.

[17] A. H. Joy, *Mt. W. Contr.*, No. 311, p. 47; *Ap. J.*, **63**, 327, 1926.

A comparison of Fowler's reproductions of the spectrum of *MgH* with the tracing of 61^2 Cygni (Fig. 5) is of great interest. The main features of the *MgH* flutings are given by an absorption tracing in the upper part of Figure 5. The agreement is so good that the identification of *MgH* in the spectrum of late-type stars must be regarded as certain. Especially noteworthy is the blue band with its first head at λ 4845, which is very easily seen in all the M dwarf spectra that I observed. It is also the strongest of the flutings found by Joy in Mira Ceti and identified by him as probably due to magnesium hydride. The strong line observed at λ 5211 coincides with the first head of the green *MgH* band.

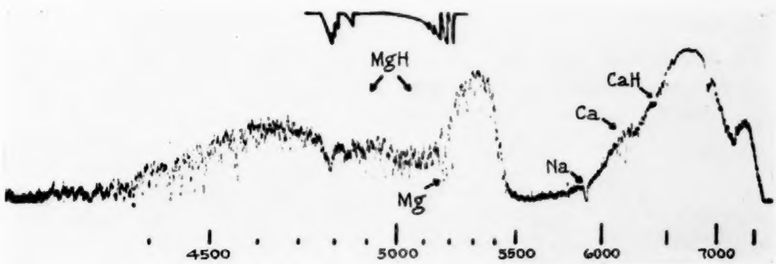

FIG. 5.—Microphotometer tracing of the spectrum of 61^2 Cygni showing bands of magnesium hydride. Above the tracing an approximate diagram of the green and blue *MgH* band systems is given. The *Ca* triplet $4^3P^\circ - 5^3S$, the λλ 6389, 6382 band of *CaH*, and the sodium D lines are marked by arrows.

Figure 5 is interesting also because of the strong calcium lines (marked *Ca*) at λλ 6162, 6122, 6102.[18] These lines have a considerable intensity in all the M dwarf spectra but are fairly weak in the giants. Unfortunately, λ 6162 is overlapped by the *TiO* band with its first head at λ 6159, but the other two lines seem very promising for the determination of absolute magnitudes, and in the case of late K-type stars and Mo stars λ 6162 could probably also be used. These three *Ca* lines are very strong in spectra of sun-spots.

It is a pleasant duty to express here my sincere thanks to Dr. W. S. Adams for his kind permission to use the 60- and 100-inch

[18] The variation of these lines with absolute magnitude was first noted by Miss Burwell (*Pub. A.S.P.*, **42**, 351, 1930).

telescopes during my stay at the Mount Wilson Observatory. I am indebted also to many other members of the staff for valuable help and interest in my work, and especially to Dr. G. Strömberg and Mr. M. L. Humason for instructive assistance with some of the exposures.

The discussion of the material has been prepared partly during a stay at the Harvard Observatory. I am much indebted to Dr. Harlow Shapley as well as to several of the other Harvard astronomers for kind interest in my work.

I have had the opportunity also to discuss some of my results with Dr. H. N. Russell, to whom I am very grateful for valuable suggestions in connection with the work on the identification of the *CaH* and *MgH* bands.

Carnegie Institution of Washington
Mount Wilson Observatory
Harvard College Observatory
March 1934

THE ORBITS OF THREE K-TYPE SPECTROSCOPIC BINARIES*

By WILLIAM H. CHRISTIE

ABSTRACT

The orbital elements of three K-type spectroscopic binaries, corrected by least-squares solutions, are given. Among these are the elements of a dwarf star, the spectra of both components of which are measurable.

BOSS 1953, γ CANIS MINORIS

The binary character of γ Canis Minoris was announced by Reese in 1902.[1] Measures of thirteen spectrograms of this star appear in

TABLE I

ELEMENTS OF THREE SPECTROSCOPIC BINARIES

Elements \ Star	Boss 1953	Boss 2824	Boss 6129
α (1900)	$7^h22^m.8$	$10^h32^m.2$	$23^h47^m.5$
δ (1900)	$+9°08'$	$34°36'$	$+74°59'$
Magnitude	4.60	6.65	6.55
Absolute magnitude	+0.1	+0.3	+6.6
Type	K4	K2	dK5
P(days)	389.0	1510.0	7.75310
T(J.D.)	2399999.53	2427408.4	2420001.264
e	0.31	0.65	0.00 (assumed)
$\hat{\omega}$	107°.4	37°.6	....
K_1 (km/sec.)	18.57	9.22	39.88
K_2 (km/sec.)	....	....	49.70
γ (km/sec.)	+46.80	+12.62	+1.68
$a_1 \sin i$ (km)	94,400,000	145,000,000	4,250,000
$a_2 \sin i$ (km)	....	....	5,300,000
$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2}$	0.26☉	0.054☉	....
$m_1 \sin^3 i$	....	....	0.321☉
$m_2 \sin^3 i$	....	....	0.258☉

Publications of the Lick Observatory, **16**, and, after an inspection of this list, the writer decided to undertake the investigation of the orbit, for, in addition to the thirteen Lick velocities, three Bonn and

* *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 499.

[1] *L.O.B.*, **1**, 159, 1902.

three Mount Wilson velocities were available. Trials for the period were made before further plates were obtained and, when the plate of December 8, 1912, was neglected, a period was readily found which satisfied all the other observations. A few plates were then obtained here, measures of which also fitted the provisional period. Suspecting that the discarded observation was in error, I asked Dr. Moore to check the data. He very kindly did so and found an error in the identification of the star on this date and that the spectrum was evidently that of ϵ Canis Majoris.[2] At the same time he thoughtfully sent me seven additional unpublished velocities of the star.

TABLE II

OBSERVATIONAL DATA; BOSS 1953

Plate	Date	Julian Date	Vel.	Phase
			km/sec.	
Lick	1900 Oct. 30	2415323.06	+43.6	152^{d}5
Lick	1901 Nov. 6	5695.99	46.8	136.5
Lick	Dec. 22	5741.93	48.8	182.4
Lick	Dec. 30	5749.84	50.4	190.3
Lick	1903 Dec. 8	6457.00	36.4	119.5
Lick	Dec. 24	6473.93	41.0	136.4
Bonn	1912 Feb. 11	9444.62	46.1	384.1
Lick	Dec. 18	9755.82	64.1	306.3
Lick	Dec. 23	9760.02	63.8	310.5
Lick	1913 Jan. 2	9770.78	65.6	321.3
Lick	Jan. 27	9795.86	58.3	346.4
Lick	Jan. 31	9799.76	55.3	350.3
Lick	Feb. 17	9816.84	49.2	366.7
Bonn	Feb. 18	9817.60	54.7	368.1
Lick	Mar. 3	9830.80	46.2	381.3
Bonn	Mar. 12	9389.52	46.9	1.0
Lick	Apr. 10	9868.69	25.4	30.2
γ 15383	1927 Nov. 6	2425191.04	59.6	295.5
15643	1928 Mar. 21	5327.65	29.2	43.1
15644	Mar. 21	5327.66	28.3	43.2
Lick	Nov. 9	5560.04	63.1	275.5
Lick	Nov. 20	5570.95	65.1	286.5
Lick	Nov. 30	5580.92	64.0	296.4
Lick	1929 Dec. 1	5946.96	62.4	273.5
Lick	1930 Feb. 17	6024.79	58.3	351.3
γ 19853	1933 Sept. 6	7323.03	31.8	93.5
19868	Sept. 8	7325.01	34.1	95.5
19959	Oct. 10	7356.96	41.4	127.5
20008	Oct. 31	7377.93	45.5	148.4
20162	Jan. 29	7467.63	53.4	238.1
20166	Jan. 29	7467.82	+52.6	238.3

[2] *Pub. A.S.P.*, **45**, 311, 1933.

With these additional observations the period was well established and the data sufficient for a least-squares solution for the most probable elements of the orbit.

The observational data are given in Table II. After rejection of the three Bonn plates, the remaining twenty-eight observations were grouped according to phase into thirteen normal places, each weighted according to the number and quality of the observations

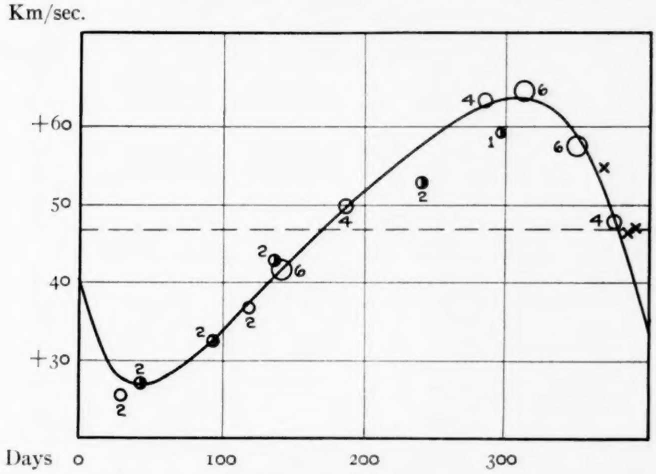

FIG. 1.—Velocity-curve of Boss 1953. Half-filled circles, Mount Wilson observations; open circles, Lick observations; crosses, Bonn observations. Numerals indicate number of observations in group.

included. All Lick plates were weighted unity; those of Mount Wilson, $\frac{1}{2}$, after a correction of -0.5 km/sec. had been applied to reduce them to the Lick system.[3] Since thirty-one orbital revolutions of the star had taken place between the first and last observations, the period was not included in the solution. The final elements are given in Table I; the corresponding velocity-curve in Figure 1. The sum of the squares of the residuals was reduced from 1464 to 958, or about 32 per cent; the probable error of a normal place of weight unity is ± 0.58 km/sec.

BOSS 2824

The binary character of Boss 2824 was announced by H. H. Plaskett in 1921.[4] The investigation of the orbit was commenced by

[3] *Pub. Lick Obs.*, **18**, xii, 1932. [4] *Pub. Dom. Ap. Obs.*, **1**, No. 26, 1922.

TABLE III

OBSERVATIONAL DATA; BOSS 2824

Plate	Date	Julian Date	Vel.	Phase
Victoria:		2420000+	km/sec.	
3746	1920 Feb. 24	2329.9	+13.1	961.d5
5813	1921 Mar. 29	2778.9	23.1	1410.5
5863	Apr. 5	2785.9	25.0	1417.5
5899	Apr. 9	2788.8	19.0	1420.4
5979	May 3	2813.8	27.6	1445.4
11329	1925 Mar. 18	4227.8	23.3	1349.4
11408	Apr. 6	4247.9	21.3	1369.5
11435	Apr. 8	4249.8	19.5	1371.4
11466	Apr. 10	4251.9	16.8	1373.5
13382	1926 May 12	4647.6	10.6	259.2
13403	May 20	4655.8	10.6	267.4
13440	June 2	4668.7	5.9	280.3
13453	June 3	4669.7	7.0	281.3
13471	June 7	4673.7	7.2	285.3
13486	June 10	4676.7	8.4	288.3
Mt. Wilson:				
γ 16577	1929 May 19	5751.6	19.1	1363.2
16590	May 21	5753.7	18.6	1365.3
C 5204	June 14	5777.7	21.6	1389.3
γ 16966	Oct. 9	5894.0	22.5	1505.6
16974	Oct. 10	5895.0	22.9	1506.6
17066	Nov. 14	5931.0	17.8	32.6
17077	Nov. 15	5932.0	17.4	33.6
17108	Nov. 22	5939.0	11.0	40.6
17163	Dec. 22	5969.0	9.9	70.6
17171	Dec. 23	5970.0	8.5	71.6
17190	1930 Jan. 19	5997.0	8.5	98.6
17195	Feb. 5	6012.9	9.6	114.5
17203	Feb. 6	6013.9	8.6	115.5
17333	Mar. 17	6054.7	10.9	156.3
17347	Mar. 18	6055.9	10.7	157.5
17388	Apr. 15	6082.6	10.0	184.2
17431	May 10	6107.6	5.7	209.2
17466	May 19	6116.7	5.8	218.3
17469	May 19	6116.8	5.6	218.4
17497	June 7	6135.7	5.5	237.3
17566	July 6	6164.7	10.0	266.3
17582	July 13	6171.7	13.3	273.3
17810	Oct. 13	6263.0	5.4	364.6
17841	Nov. 1	6283.0	9.0	384.6
17848	Nov. 2	6284.0	4.1	385.6
17907	Nov. 29	6310.0	7.5	411.6
17916	Nov. 30	6311.0	3.6	412.6
17968	Dec. 9	6320.0	12.1	321.6
17984	Dec. 12	6323.0	12.6	424.6
17986	Dec. 24	6335.8	17.0	437.4
17988	Dec. 24	6336.0	12.8	437.6
17991	Dec. 26	6337.9	14.9	439.5
18078	1931 Mar. 1	6402.8	0.3	504.4
18124	Mar. 9	6410.8	+ 7.6	512.2

TABLE III—*Continued*

Plate	Date	Julian Date	Vel.	Phase
Mt. Wilson—*Continued:*		2420000+	km/sec.	
γ 18138	1931 Mar. 13	6414.8	+ 6.8	516ᵈ4
18158	Mar. 30	6431.6	8.2	533.2
18168	Mar. 31	6432.9	10.2	534.5
18184	Apr. 5	6437.8	5.8	539.4
18210	Apr. 30	6462.7	7.8	564.3
18223	May 3	6465.8	10.6	567.4
18281	June 3	6496.7	14.0	598.3
18555	Nov. 17	6664.0	9.0	765.6
18560	Nov. 18	6665.7	12.8	767.3
18631	Dec. 30	6707.0	12.8	808.6
18685	1932 Jan. 29	6736.8	14.8	838.4
18692	Feb. 19	6757.8	12.1	859.4
18698	Feb. 20	6758.7	10.8	860.3
18758	Mar. 22	6789.7	8.6	891.3
18759	Mar. 23	6790.6	9.3	892.2
18809	Apr. 21	6819.8	7.1	921.4
18811	Apr. 22	6820.8	7.7	922.4
18855	May 24	6852.7	8.5	954.3
18921	June 25	6885.7	12.9	987.3
19184	Oct. 10	6992.0	10.7	1093.6
19238	Nov. 6	7019.0	10.7	1120.6
19259	Nov. 8	7021.0	16.3	1122.6
19343	Dec. 5	7048.0	13.2	1149.6
19359	Dec. 6	7049.0	14.3	1150.6
19536	1933 Mar. 9	7141.6	16.9	1243.6
19593	Apr. 9	7172.6	19.4	1274.2
19649	May 14	7207.6	20.5	1309.2
19657	May 15	7208.6	18.2	1310.2
19663	May 31	7224.7	24.4	1326.3
19971	Oct. 11	7358.0	28.2	1459.6
20010	Oct. 31	7378.0	24.8	1479.6
20039	Nov. 9	7387.0	24.8	1488.6
20047	Nov. 10	7388.0	23.8	1489.6
20071	Dec. 1	7409.0	22.5	0.6
20117	1934 Jan 2	7441.0	15.4	32.6
20168	Jan. 29	7467.9	13.5	59.5
20202	Feb. 2	7471.8	+ 5.3	63.4

Pearce in 1925, but at his suggestion was continued by me while serving as observing assistant at the Dominion Astrophysical Observatory during the summer of 1926. In all, eighty-seven usable spectra were taken before the preliminary elements could be determined with any degree of accuracy, sixteen at Victoria, the remainder at Mount Wilson. The observational data are given in Table III.

The preliminary period of 1510 days was not definitely ascertained until the end of 1933, and some difficulty was encountered in deter-

mining the preliminary elements because of peculiar irregularities in the run of the individual observations. These irregularities suggest a secondary variation, in a period of about 220 days, superimposed upon the longer period. The individual observations from 1929 to 1932, inclusive, decidedly favor such a variation. Since the amplitude of this secondary variation, if real, amounts to only 7 km/sec., higher dispersion than that of the one-prism spectrographs used throughout this investigation is necessary to settle the question;

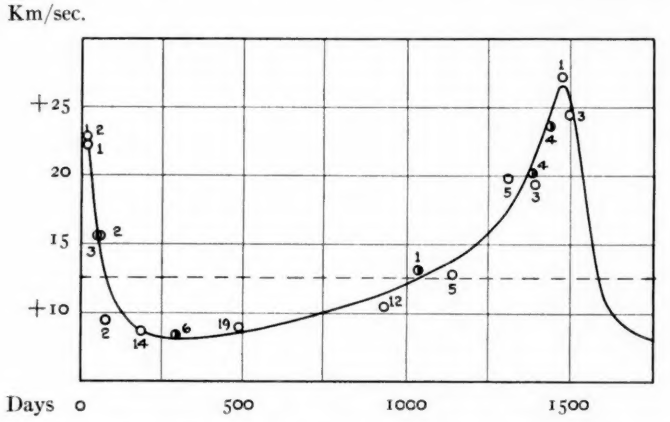

FIG. 2.—Velocity-curve of Boss 2824. Open circles, Mount Wilson observations; half-filled circles, Victoria observations. Numerals indicate number of observations in group.

but since the star is not bright, the time consumed by such a procedure would be prohibitive.

In order to include the Victoria plates in the solution, they were remeasured here and reduced by the use of the Mount Wilson tables; hence, unless there is a large systematic instrumental effect, the material should be fairly homogeneous.

The eighty-seven observations were grouped according to phase into eighteen normal places, weighted according to the number of observations in each group. After satisfactory preliminary elements had been derived from the data, a solution was made for the most probable values. The resulting corrected elements are given in Table I, the corresponding velocity-curve in Figure 2. The probable error of a normal place of weight unity is ± 0.38 km/sec. The sum

of the squares of the residuals was reduced from 1018 to 807, about 21 per cent.

BOSS 6129

The binary character of Boss 6129 was announced in 1917 by Adams and Joy.[5] A spectrogram of this star taken by the writer in

TABLE IV

OBSERVATIONAL DATA; BOSS 6129

Plate	Date	Julian Date	Vel. Br.	Vel. Ft.	Remarks
			km/sec.	km/sec.	
γ 2888...	1913 Nov. 15	2420087.842	+23.2	....	Shows traces of doubling
3894...	1914 Nov. 24	0461.725	+16.7	....	Single lines; omitted
4293...	1915 July 28	0707.983	+27.2	....	Shows traces of doubling
4928...	1916 July 12	1056.943	+41.4	−41.8	
4995...	Aug. 16	1091.962	−43.1	+48.8	
5089...	Sept. 17	1124.842	+11.9	....	Questionable; omitted
5427...	1917 Jan. 6	1235.626	+51.6	−51.9	
19677...	1933 June 1	7225.973	−31.2	+45.1	
19692...	June 9	7233.975	−24.0	+37.7	
19699...	June 10	7234.985	+ 3.6	....	
19710...	June 30	7254.983	−16.7	+27.8	
19748...	July 12	7266.950	+27.4	....	
19756...	July 13	7267.909	+44.7	−61.1	
19797...	Aug. 11	7296.877	+ 7.1	....	
19809...	Aug. 12	7297.959	+28.9	....	
19815...	Aug. 13	7298.896	+38.7	−40.2	
19849...	Sept. 6	7322.846	+35.2	−33.1	
19858...	Sept. 7	7323.835	+ 8.6	....	
19963...	Oct. 10	7356.802	−44.4	+57.6	
19954...	Oct. 11	7357.729	−27.8	+35.1	
20016...	Nov. 2	7379.907	−32.1	+48.6	
20034...	Nov. 9	7386.677	−13.9	+26.7	Blend +3.1 km/sec.
20041...	Nov. 10	7387.631	−38.1	+35.0	

the summer of 1933 showed double lines. On looking over the seven older plates, taken between 1912 and 1918, three were found on which the lines were distinctly double, while on two others they showed definite traces of duplicity. As double-line spectroscopic binaries of type K or later are extremely rare (the companion to Castor being the only other example known to the writer), the star

[5] *Pub. A.S.P.*, **29**, 114, 1917.

was placed on the program for further observation. Eleven of the fourteen spectrograms obtained during the summer and fall of 1933 showed the spectra of both components sufficiently separated to be measured with confidence.

Except for an intensity difference of about 1 mag., the two spectra appear to be quite similar. According to Adams, Joy, and Humason, the spectral type and absolute magnitude of the brighter star are dK5 and 6.5, respectively. The trigonometric parallax, kindly fur-

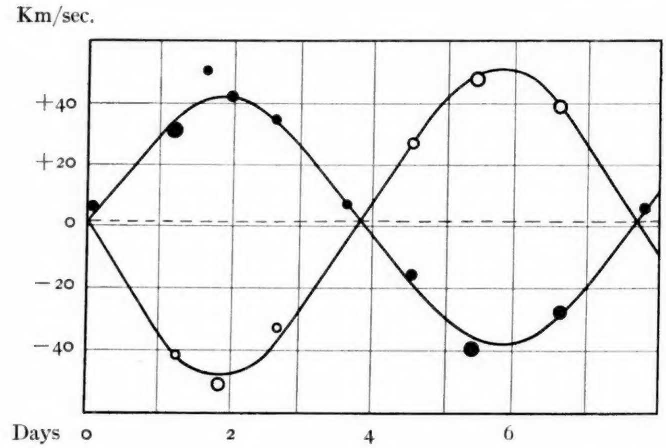

FIG. 3.—Velocity-curve of Boss 6129. Size of circles proportional to weights

nished by van Maanen, is $0\rlap{.}''090 \pm 0\rlap{.}''004$, which, with an adopted visual magnitude of 6.8 for the primary, gives an absolute magnitude of +6.6, in excellent agreement with the spectroscopic value. The secondary spectrum is somewhat difficult to measure, and the scatter for the individual lines is rather large. Nevertheless, these results appear to be quite reliable, for the errors are no greater than may be expected. Even when the lines of the two spectra overlap, it has been found possible to measure them with some accuracy.

The period was found without difficulty and, because of the twenty-year interval between the first and last observations, there was no necessity of including it in the least-squares solution. The observational data are given in Table IV. The observations of the primary, with the exceptions noted in the "Remarks" column of the

table, were grouped according to phase into nine normal places, each weighted according to the number of observations included.

As no eccentricity was apparent, this element was assumed to be zero, and a least-squares solution was made for γ, K, and T. The secondary was not included in the solution because of the small weight assigned to the measures, but, by using the final values of γ and T derived from the primary, a correction was made to the semi-amplitude of the secondary. The final elements are given in Table I, the velocity-curve in Figure 3.

The probable error of a normal place of weight unity is ± 0.39 km/sec. for the primary and ± 1.13 km/sec. for the secondary. The sum of the squares of the residuals for the primary was reduced from 380 to 234, and for the secondary from 265 to 81, or about 39 and 70 per cent, respectively.

Carnegie Institution of Washington
Mount Wilson Observatory
April 1934

IDENTIFICATION OF LINES IN THE SPECTRA OF B STARS

By P. SWINGS and M. NICOLET

ABSTRACT

Identifications of lines in the spectra of B stars are given. About fifty lines for which no previous identifications had been made are interpreted.

Within recent years three important lists of absorption lines in B stars have been published[1] by O. Struve;[2] by O. Struve and T. Dunham, Jr.;[3] and by Roy K. Marshall.[4] The first list deals with spectra extending from λ 3850 to λ 4923;[5] the ten stars investigated are of spectral types O9, B0, B1, B2, B3, B5, B8. The plates were taken either with the Coudé spectrograph of the Mount Wilson Observatory or with the Bruce spectrograph of the Yerkes Observatory, on Eastman Process emulsion, which gives excellent contrast. Of the total number of lines measured, 73 remained unidentified. The list by O. Struve and T. Dunham, Jr., deals only with the spectrum of the B0 star 23 τ Scorpii. The spectrogram was obtained with the Coudé spectrograph and the 100-inch Mount Wilson reflector, on Eastman Process emulsion, and extends from about λ 3945 to λ 4713. Several new identifications for *C* III, *N* III, and *O* III were added to the first list of Struve, while 53 lines remained unidentified. The study of class B stellar spectra made by R. K. Marshall is based on spectra taken with the single-prism spectrograph attached to the $37\frac{1}{2}$-inch reflector of the University of Michigan. Owing to the unusually high transmission of that optical system, Marshall's list begins at λ 3587.19 A. Most of the plates were taken on Eastman Process emulsion. Of the 534 lines tabulated, almost exactly one-half have not been identified.

[1] The following abbreviations will be used: O. S. (O. Struve), S. D. (Struve-Dunham) and M (R. K. Marshall).

[2] *Ap. J.*, **74**, 225, 1931.

[3] *Ibid.*, **77**, 321, 1933.

[4] *Pub. of the Observatory of the University of Michigan*, **5**, No. 12, 137, 1934.

[5] The region $\lambda\lambda$ 3850–3950 is given only for γ Pegasi (B2); for wave-lengths longer than 4700, only the stronger lines could be measured.

Marshall's list of ultra-violet lines is certainly excellent, but in the ordinary photographic region it is inferior to those of Struve and of Struve and Dunham.

In the following paper we give a few additional identifications. Attention should be drawn toward the following points:

1. In each multiplet the relative intensities of the stellar lines must be compatible with the laboratory intensities.

2. The absolute laboratory intensities have to be considered.

3. The spectral type corresponding to maximum intensity of the stellar line and the behavior of intensity as a function of spectral type must be reasonable.

4. The gradient effect makes all weak absorption lines in dwarfs of type B relatively stronger than the laboratory intensities might indicate.

5. The atomic weight of the elements and their normal abundances have to be considered.

6. It may happen that for two different spectral types—for example, Bo and B8—lines of the same measured wave-length may have different origins.

7. The difference λ obs. $-\lambda$ lab., which is allowed for an identification, depends upon the dispersion in that region and upon the structure of the line (intensity, sharpness).

8. It may happen that a faint line has been omitted in the measures of one particular star.

In this paper none of the lines recently identified by R. K. Marshall (*O* III),[6] O. Struve (*C* III),[7] J. E. Mack, P. Swings, and O. Struve (*C* IV),[8] O. Struve and H. Pillans (*Si* IV),[9] D. H. Menzel and R. K. Marshall (*Ne* II);[10] and E. Mac Cormack (*Ne* I)[11] has been listed. When a line of one of the three lists considered here has been identified in one of the other lists, it has been omitted here. When components of a multiplet have been identified by O. S., S. D., or M, the identification of other lines in the same multiplet is comparatively easy; only the new identifications are then given, the symbol of the multiplet being written in brackets.

[6] *Ap. J.*, **76**, 317, 1932.

[7] Included in list S. D.

[8] *Ap. J.*, **75**, 77, 1932.

[9] *Observatory*, **57**, 133, 1934.

[10] *Proc. Nat. Acad.*, **19**, 879, 1933.

[11] *Pub. A.S.P.*, **46**, 64, 1934.

The authors believe that an investigation of the ultra-violet region of $\lambda < 3600$ A (by using an aluminized telescope and a quartz or U.V spectrograph) would be fruitful.

TABLE I

C II

Multiplet Designation	Laboratory Wave-Length	Laboratory Intensity	Stellar Wave-Length	Author	Spectral Types	Identification
$2p''^2P-3p'^2S$	3868.84	2	8.81	M	B2	XXX
	3871.62	1	1.89	M	....	Bl. *He* I
$(3d'^4F-4f^4G)$	3878.22	1	8.10	M	B2	Bl. *He* I
	79.60	1	0.38	M	B1, 2	XXX?
	80.59	1				
$(3d'^4D-4f'^4F)$	4076.00	7	5.88	O.S.	B0, 1, 2	Bl. *O* II
$3d'^2D-4f'^2D$	4285.96	1	6.24	O.S.	B2	XXX
	96.11	1	6.54	O.S.	B0, 2	XXX
$3p'^4P-4s'^4P$	4313.50	2	3.32	O.S.	B0, 1, 2	Bl. *O* II
	17.42	4	7.21	O.S.	B0, 1, 2	Bl. *S* II
	18.92	2	8.45	O.S.	B0, 2	Bl. *O* II
	25.88	2	5.64	O.S.	B0, 1, 2	Bl. *O* II
Absent	4630.52	1	0.55	O.S.	B0, 1, 2	Bl. *O* II

Less probable identifications ($\Delta\lambda$ seems too large):

No designation	4292.00	1	2.55	S.D.	B0	Bl. *O* II
	4376.78	1	6.23	M	B0	Bl. *O* III
	4625.71	1	5.16	O.S.	B1, 2	XXX

TABLE II

N II

Multiplet Designation	Laboratory Wave-Length	Laboratory Intensity	Stellar Wave-Length	Author	Spectral Types	Identification
$(3d^3D-4f^3D)$	4160.8	0	0.62	M	B1, 3	XXX
	73.51	0	3.59	O.S.	B2, 3	Bl. *S* II
	73.75	0	4.12			
			4.06	M	B2, 3	Bl. *S* II
$3d^1D-4f^1D$	4044.75	1	5.00	O.S.	B2	XXX

C II, N II, N III, and O II.—The identifications are given in Tables I, II, III, and IV.[12]

TABLE III

N III

Multiplet Designation	Laboratory Wave-Length	Laboratory Intensity	Stellar Wave-Length	Author	Spectral Types	Identification
($3s'^2P-3p'^2D$). .	4215.69	3	5.79	S.D.	Bo	Bl. [*N* II]
$4p^2P-5s^2S$.	4544.80	o	4.90	S.D.	Bo	XXX

Less probable identifications ($\Delta\lambda$ seems too large)

Multiplet Designation	Laboratory Wave-Length	Laboratory Intensity	Stellar Wave-Length	Author	Spectral Types	Identification
$3p'^2D-3d'^2D$. .	3934.41	3	3.62	M	Bo	Bl. *Ca* II, *S* II
	38.52	4	9.12	M	Bo	XXX?
	42.78	I	3.46	M	Bo	XXX?

TABLE IV

O II

Multiplet Designation	Laboratory Wave-Length	Laboratory Intensity	Stellar Wave-Length	Author	Spectral Types	Identification
$4p^2P-4d^2D$. . .	3785.01	o	5.36	M	B1, 2	XXX
$4s^2P-4p^2D$. . . .	3838.41	o	8.30	M	B1, 2	Bl. *He* I, *N* II
($3d^4F-4f^4F$). . . .	4026.40	o	6.31	S.D.	Bo	Bl. *He* I, *N* II
	33.18	o	2.90	O.S.	B2	XXX
	44.96	o	5.00	O.S.	B2	XXX
	46.15	o	5.80	M	Bo	XXX
$e^4P^o-z^4D$.	4103.01	5	3.40	O.S. and M	O9, Bo, 2	Bl. *N* III
$3d^2D-4f^2D$. . .	4342.83	I	2.90	O.S.	Bo, 1	Bl. *O* II
($3d^2D-4f^4D$). . .	4482.85	o	2.98	S.D.	Bo	Bl. *O* III
($3d^2D-4f^2D$). . .	4613.67	I	3.84	S.D.	Bo	Bl. *N* II
	21.28	o	1.49	S.D.	Bo	Bl. *N* II
($3d^2D-4f^2D$). . .	4707.80	o	7.98	O.S.	Bo, 1, 2	XXX

[12] In these tables the symbol xxx means that no identification existed previously for the line; in case of blending, the blending line is indicated.

Ne II.—A few remarks may be made concerning the identification of *Ne* II lines by D. H. Menzel and R. K. Marshall.[10]

1. In $3s^4P-3p^4P^o$ it does not seem possible that the strong *Ne* II line λ 3766.29 which is at a distance of 4 *A* from *Hι* could have been masked by *Hι*.

2. In $3p^2P^o-3d^2D$ the identifications are rather doubtful, as the strongest line λ 3829.77 does not appear in 10 Lacertae (an O9 spectrum with very numerous and sharp lines), although the region from λ 3820 to λ 3835 in 10 Lacertae is completely free from lines.

3. In $3d^4D-4f^4D^o$ three lines by O. S. and S. D. are identified by Menzel and Marshall; it is also possible to identify λ 4217.22 (τ Scorpii, Bo, int. 2) with λ lab. 4217.15 (int. 3).

4. In $3d^4F-4f^4G^o$ two identifications may be added to the two made previously by Menzel-Marshall:

Multiplet Designation	Laboratory Wave-Length	Laboratory Intensity	Stellar Wave-Length	Author	Spectral Types	Identification
$(3d^4F-4f^4G^o)$...	4290.40	6	{0.53 0.47	S.D. M	Bo Bo	Bl. *N* III
	4428.54	6	{8.53 8.44	S.D. M	Bo Bo	Bl. [*N* II]

5. In $3d^4F-4f^4F^o$ other identifications in τ Scorpii (S.D.) are allowed:

$(3d^4F-4f^4F^o)$...	4369.77	5	9.33	S.D.	Bo	Bl. *O* II
	4397.94	6	8.01	S.D.	Bo	XXX
	4430.90	4	0.99	S.D.	Bo	Bl. [*S* II]
	42.67	3	2.95	S.D.	Bo	Bl. *O* II
	46.46	3	6.97	S.D.	Bo	Bl. *N* II, *O* II

6. An identification may be made in $3d^2P-4f^2D^o$:

$3d^2P-4f^2D^o$...	4511.29 11.37	2 4	11.13 10.88	M S.D. and O.S.	Bo O9, Bo	Bl. *N* III

Na II.[13]—The *Na* II absorption lines having a wave-length greater than 3580 A require an excitation potential of more than 32 volts; thus they should be strong only in the hottest stars. λ 3631.37 (int. 8) of *Na* II may tentatively account for λ 3631.67 (class O9). An examination of the ultra-violet parts of a few O5–O8 spectra would be interesting.

Na III and Mg III.—The lower excitation potentials of the visible lines are:

For *Na* III : ≥ 57 volts ,

For *Mg* III : ≥ 65 volts .[14]

Thus it seems impossible to expect these lines in B-type spectra.

Al II and III.—Marshall casts suspicion upon the identification of *Al* II and *Al* III. The identification of *Al* III is absolutely safe and some of the lines are actually strong (e.g., λλ 4150, 4480, 4512.5, and 4529), as may be seen on the illustrations of the spectrum of γ Pegasi given in Struve's complete paper.[15]

Besides λλ 3601.73 (int. 4) and 3612.26 (int. 2) observed by Marshall in α Pegasi and β Canis Majoris and which are certainly due to *Al* III, two lines in the visible part of Marshall's list, λλ 4150.05 and 4529.45, have their origin in *Al* III.

Al II seems also to be definitely present. Several lines in Marshall's list may be attributed to *Al* II by comparison with Struve's complete list. In addition to those lines a few more identifications are indicated in Table V.

Table V shows that *Al* II is certainly present in B stars. It may be recalled that in the Sun the abundances of *Al* and *Si* are of the same order ($\mu_{Al} \sim 10^{-1} \mu_{Si}$). The three first ionization potentials are similar:

Al I 5.96 ; *Al* II 18.73 ; *Al* III 28.31 ;

Si I 8.12 ; *Si* II 16.27 ; *Si* III 33.30 .

P III and IV.—For *P* III the identifications are indicated in Table VI.

[13] Questioned by Marshall, *op. cit.*, p. 153.

[14] Kindly communicated by Dr. B. Edlén and Dr. J. Söderqvist.

[15] *Op. cit.*, p. 248, Pl. XI.

TABLE V

Al II

Multiplet Designation	Laboratory Wave-Length	Laboratory Intensity	Stellar Wave-Length	Author	Spectral Types	Identification
$(4^3P^o-5^3D)$.....	3649.20	1.5	9.58	M	B2	XXX
	51.06	6	1.14	M	B2	XXX
$4^1P^o-6^1D$......	3703.22	4	3.38	M	B2	XXX
4^3D-12^3F......	3734.7	1	4.33	M	....	Bl. H_λ
4^3P-6^3S.......	3731.95	1	2.04	M	B2, 3	XXX
	3.91	2	4.33	M	....	Bl. H_λ
	8.00	3	{7.87	L*	B3	XXX
			7.35	M	B5	XXX
5^1S-12^1P......	3753.10	1	3.44	M	B2, 8	XXX
5^3S-8^3P.......	3774.3	0	3.73	M	B2, 3, 5	XXX?
4^3D-11^3F......	3842.2	3	2.74	M	B5	Bl. *O* II, *N* II?
$(4^3D-10^3F^o)$....	3996.08	1	6.66	M	B5	XXX
	.16	4				
	.32	0.5				
	.38	3				
5^1S-10^1P......	4009.58	1	9.32	M	B2, 3, 5, 8	Bl. *He* I

* Measured by H. M. Losh, in ζ Tauri (*Pub. Obs. U. of Michigan*, **4**, 1, Table 10, 1931).

TABLE VI

P III

Multiplet Designation	Laboratory Wave-Length	Laboratory Intensity	Stellar Wave-Length	Author	Spectral Types	Identification
$4s^4P^o-4p^4P$....	3895.03	6	5.56	M	B1, 2	XXX
	3904.79	6	4.88	M	B0, 1	XXX
	22.72	4	2.79	M	B0, 1	XXX
	33.38	4	3.62	M	B0, 1	Bl. *Ca* II, *S* II
	51.51*	5	2.75	M	B1	XXX
	57.64	6	7.54	M	B0, 1, 2	XXX
	97.17	5	6.66	M	B0, 1	XXX

* This line is quoted by Marshall as being of poor quality; the wave-length was not measured in the B1 star β Cephei, but in the B5p star 67 Ophiuchi; the lines in β Cephei and in 67 Ophiuchi have probably different origin.

P behaves similarly to *Si*, as is shown by the ionization potentials:

P I 11.11 ; *P* II 19.81 ; *P* III 30.23 ; *P* IV 51.1 ;

Si I 8.12 ; *Si* II 16.27 ; *Si* III 33.30 ; *Si* IV 44.91 .

Bowen[16] has found a line of *P* IV at λ 4249.57 (int. 6) having the designation $4^1S-4^1P^o$ (lower E.P.-28.9 volts), which could possibly

TABLE VII

S II

Multiplet Designation	Laboratory Wave-Length	Laboratory Intensity	Stellar Wave-Length	Author	Spectral Types	Identification
$4p'^2D^o-4s^2P$....	3669.03	4	9.23	M	B2	XXX
$4p^2S^o-5s^2P$.....	3783.13	3	2.78	M	B2	XXX
$4p^4P^o-4d^4P$....	3792.46	5	2.63	M	B2, 3	XXX
	3802.68	1	3.12	M	B2	Bl. *O* II
	60.66	3	0.58	M	B2, 3, 5	XXX
	92.32	5	2.21	M	B2, 3, 5	XXX
$(4p^4D^o-4d^4D)$...	3939.52	0	9.12	M	B1, 2, 3	XXX?
	47.02	2	7.28	M	B0, 1, 2	XXX?
	50.45	0	9.67	M	B0, 1, 2, 3	XXX?
	63.14	3	2.64	M	B0, 5	Bl. *O* II
	70.68	2	0.10	M	....	Bl. *H*ε?
	90.94	8	0.98	M	B1, 2, 5	XXX
	4003.90	3	3.58	S.D.	B0	Bl. *N* III
$(4p^2D-4d^2F)$....	4009.41	2	9.28	O.S.	....	Bl. *He* I [*C* II]
$(3d^2F-4p'^2F^o)$...	3993.49	6	3.61	M	B2	XXX

explain the unidentified λ 4250.10 (int. 1) of 10 Lacertae (class O9) and λ 4249.89 (int. 1) of τ Scorpii (class B0).

S II and III.—According to Marshall,[17] Gilles's tables are untrustworthy, but the investigations of O. Bartelt and L. Eckstein[18] have brought new material which has allowed the identifications listed in Tables VII and VIII.

[16] *Phys. Rev.*, **39**, 8, 1932.

[17] *Op. cit.*, p. 152.

[18] *Zs. für Phys.*, **86**, 77, 1933; *Zs. für Astroph.*, **7**, 272, 1933.

TABLE VIII

S III

Multiplet Designation	Laboratory Wave-Length	Laboratory Intensity	Stellar Wave-Length	Author	Spectral Types	Identification
$(4s^3P^o-4p^3P)$	3837.79	5	7.33	M	B1, 2	XXX
	60.64	4	0.58	M	B1, 2, 3	XXX
$(3d^3D^o-4p^3D)$	4499.29	0	9.40	S.D.	B0	XXX?
	4527.96	0	7.18	M	B0	XXX??

Ca III.—There are two laboratory lines in the astrophysical region:

$4s_3-4p_{10}$	3761.62	6	1.27	M	O9	XXX
$4s_2-4p_{10}$*	4081.74	5	1.83	M	O9–B0	XXX

* This second identification is somewhat doubtful, as there exists some incompatibility between the measures of S.D. and M near λ 4081 A.

Ti II.—Possible identifications are indicated in Table IX.

TABLE IX

Multiplet Designation	Laboratory Wave-Length	Laboratory Intensity	Stellar Wave-Length	Author	Spectral Types	Identification
$b^2D-y^2D^o$	3741.68	(50)	1.85	M	B3	XXX
	57.69	(30)	7.70	M	B8	XXX
	76.06	(6)	5.97	M	B5–B8	XXX

A II.—When comparing all the multiplets of *A* II given by De Bruin[19] with the absorption lines of classes B0, B1. and B2, we find a percentage of coincidences as large as in the case of *Ne* II. The number of coincidences observed is obviously greater than the number of chance coincidences calculated by the Russell-Bowen formula; in addition, the intensities observed agree quite well with the laboratory intensities. If the presence of *A* II could be verified with certainty, an important number of lines of B stars would be identified;[20] these lines appear where they ought to (maximum at B1–B2), owing

[19] *Zs. für Phys.*, **48**, 62, 1928; **51**, 108, 1928; **61**, 307, 1930.

[20] More than twelve, which had heretofore no other suggested identification.

to their ionization and excitation potentials, which are lower than for *Ne* II.

Ionization potentials:

$$Ne\ \text{I}\ 21.47\ \text{volts}\ ; \quad Ne\ \text{II}\ 40.9\ \text{volts}\ ;$$

$$A\ \text{I}\ 15.69\ \text{volts}\ ; \quad A\ \text{II}\ 27.72\ \text{volts}\ .$$

Excitation potential of lower level:

$$Ne\ \text{II}\ :\ \text{between}\ 27\ \text{and}\ 28\ \text{volts}\ ;$$

$$A\ \text{II}\ :\ \text{between}\ 17\ \text{and}\ 22\ \text{volts}\ .$$

Arguments against these identifications are the atomic weight of argon (39.94) and the fact that it has never been observed in stellar or nebular spectra. These arguments do not seem convincing. The discovery of *A* II in B stars would be of interest, in connection with a recent paper by H. N. Russell and D. H. Menzel.[21]

The question of *A* II is being investigated here; the results will appear later.

University of Liége (Belgium)
Department of Astrophysics
May 10, 1934

[21] *Proc. Nat. Acad.*, **19**, 997, 1933.

SYSTEMATIC ERRORS OF PARALLAXES

By S. A. MITCHELL

ABSTRACT

A re-examination is made of the trigonometric parallaxes from eight observatories and of their differences from the revised (as yet unpublished) Mount Wilson spectroscopic parallaxes, which material was kindly furnished by Dr. van Maanen. It is concluded that the observations are sufficiently numerous to determine fairly satisfactorily the systematic error in zero point, but that in most cases there is too little material to detect real variation of the systematic error with right ascension.

In coming to this conclusion, the parallaxes from the McCormick Observatory are considered in detail. After making harmonic analyses for various samples of the material by omitting stars with large parallaxes, the curves show few resemblances to one another. In particular, when parallaxes greater than 0".050 are excluded, the curve derived from the remaining 546 stars shows almost no variation with right ascension. This curve should give the most reliable information concerning the McCormick parallaxes, since for these stars the accidental errors of the spectroscopic parallaxes may be disregarded.

Comparisons are then made between the Allegheny series and that of McCormick.

Finally, the probable errors of van Maanen's Fourier coefficients are derived for each of the eight observatories. From a consideration of these it is concluded that the periodic terms derived from the harmonic analyses have little physical significance.

The question of systematic errors of parallaxes is one of perennial interest and importance. In the last ten years there have been four extensive investigations. In order of date of publication these are: In 1925, in the *Groningen Publications*, No. 37, van Rhijn derived systematic errors for both trigonometric and spectroscopic parallaxes from comparisons with mean statistical parallaxes. In 1926, Schlesinger published the *Yale Catalogue*, containing parallaxes published up to 1924 together with systematic errors of the different trigonometric series. In 1928 and in 1933, in *Mount Wilson Contributions* Nos. 356 and 474, van Maanen discussed the systematic errors of the trigonometric parallaxes determined by photography by the various contributing observatories. Of these four discussions, the most recent one by van Maanen obviously gives the most reliable values of systematic errors, mainly for two reasons: (1) the greater number of trigonometric parallaxes available and (2) a redetermination of the Mount Wilson spectroscopic parallaxes by Adams. In spite of the very high quality of van Maanen's results, a question should be raised regarding the precision with which his tabular values actually represent the systematic errors of the

trigonometric parallaxes of the different observatories. It will be shown that the van Maanen curves reflect chiefly the effects of *accidental* errors and only in a minor way have a bearing on *systematic* errors.

All extended series of precise observations are subject to errors of two different kinds, accidental and systematic. Investigations of accidental errors gave rise to the theory of errors and the method of least squares, and as a result we have a fair degree of confidence in our ability to handle accidental errors. With systematic errors the problem is much more difficult, on account of the necessity of finding an independent basis of comparison where systematic and accidental errors are inappreciable in size.

Each of the astronomers responsible for the different trigonometric parallax series naturally admits that his parallaxes are subject to accidental errors. Each observer hopes that he has taken and measured his photographs with such minute care that there is no possibility of systematic errors in his own particular parallaxes. In the meetings of the American Astronomical Society, much good-natured fun has been made of the different parallax observers in their ability to prove, each to his own satisfaction, that his own parallaxes are the best of all. These remarks should not be taken too seriously. He would be a very unwise scientist who would insist that though his observations are subject to accidental errors they are quite free from systematic errors. Unquestionably each and every one of the trigonometric parallax series, no matter how carefully made, is subject to systematic errors, some series having smaller errors than others, but the problem of finding adequate values of these systematic errors is a different story. For example, Allegheny and McCormick each began parallax work in the fall of 1914; van Maanen began at Mount Wilson about a year earlier. The Allegheny and McCormick programs have had so many stars in common that up to the present one-half of all the stars published by either observatory is common to the other. With the van Maanen program there has not been an equal overlapping. In the first decade of parallax work, in comparing McCormick parallaxes with those of Allegheny and Mount Wilson, it was found on the average that the McCormick parallaxes were systematically larger than those of Allegheny and system-

atically smaller than those of Mount Wilson. In the second decade of work, both Schlesinger and I have noted independently that every new list of parallaxes published by either Allegheny or McCormick showed no further evidence of systematic differences. Apparently one or the other, or both observatories, had changed their habits of observing, with the result that the causes of the systematic differences were eliminated. At one time the difference of Allegheny − McCormick was $-0\rlap{.}''004$; now this difference has been greatly decreased until it is $-0\rlap{.}''0015$ or less. Evidently the systematic differences between Allegheny and McCormick (as will be shown on p. 221) are not the same in the second decade of observations as they were in the first. If, therefore, one is to insist on an exact definition of terms, the differences are not "systematic" or constant. Hence, if Allegheny and McCormick continue with their present methods of observation, it appears probable that the systematic differences between their parallaxes will become less and less as more and more observations are accumulated.

The method adopted by Schlesinger in deriving the systematic errors found in the Yale catalogue is fundamentally the same as was followed by van Maanen in his discussions, each of the three derivations differing only in details from the other two. The Mount Wilson spectroscopic parallaxes were used as the basis of comparison. After reducing the trigonometric relative parallaxes to absolute values, the differences between the spectroscopic and trigonometric parallaxes for each star were taken for each of the trigonometric parallax series. For each series means were taken for each of the twenty-four hours of right ascension. Up to this point in the investigations, Schlesinger and van Maanen followed practically the same procedure. The former has not published the details of the further steps, although it is not difficult to make a guess regarding the technique followed. In the two *Mount Wilson Contributions*, van Maanen has explained his methods in detail. After forming the hourly means of the differences between trigonometric and spectroscopic parallaxes, he has rounded off the hourly means by taking the weighted means of three successive hours of right ascension. In the discussion of 1928, by the use of harmonic analysis with five constants, and in 1933 by similar methods but with seven constants, van Maanen derived by

least squares values for each trigonometric parallax series, which were called the "systematic corrections of the trigonometric parallaxes." As these values varied with right ascension, it was assumed that they were the result of seasonal effects.

I myself have never been satisfied that the corrections published by Schlesinger and by van Maanen give what they purport to give, namely, the systematic errors of the various trigonometric series. Through good fortune, not long after the publication of van Maanen's discussion of 1928, I was in residence at the Mount Wilson Observatory for two months. As a result of discussions with him and with other members of the Mount Wilson staff, van Maanen kindly gave to me the individual differences, Trigonometric – Spectroscopic, that he had tabulated for the two longer series of parallaxes, Allegheny and McCormick. The results of my own discussion of his material were reported at the New Haven meeting of the American Astronomical Society in 1929 and again in Commission 24 of the International Astronomical Union in 1932. On pages 246–247 of the *Transactions*, brief reference is made to my conclusions. Following the publication of van Maanen's 1933 discussion, I wrote to him, and again with the very greatest of kindness he sent to me all of the differences, Trigonometric – Spectroscopic, for each observatory making parallax measurements. I am under a great debt of obligation to him. In all that follows I have confined my discussion chiefly to two observatories, Allegheny and McCormick, for the reasons that the parallaxes are more numerous and the published values of the systematic errors are smaller than for the other observatories. For obvious reasons, I have gone into greater detail with the McCormick parallaxes.

In forming the difference between the trigonometric parallax (π_t) and the spectroscopic parallax (π_s) the difference $\pi_t - \pi_s$ is taken for each star for each observatory. In assuming that the values and curves which are functions of right ascension actually represent the systematic errors of the trigonometric series, both Schlesinger and van Maanen tacitly make the following assumptions regarding the four classes of errors, systematic and accidental, that enter into both trigonometric and spectroscopic parallaxes: (1) The spectroscopic parallaxes are so numerous, now amounting to four thousand,

and it is unbelievable that they are subject to seasonal effects or have any dependence on right ascension. With such a large number of spectroscopic parallaxes it is quite reasonable to assume that they may possibly have a zero-point correction but that they are devoid of systematic errors depending on the star's position in the heavens. (2) The accidental errors of the spectroscopic parallaxes, and (3) the accidental errors of the trigonometric parallaxes have a habit of smoothing themselves out in such a fashion that there is no possibility of a systematic trend depending on right ascension. The final results from the discussions of the differences are then assumed (4) to represent the systematic errors of the trigonometric parallaxes, probably correct to an accuracy of $0''.001$. It will be shown in what follows that not one of these four assumptions is true.

It is my opinion that the Mount Wilson spectroscopic parallaxes, soon to be fully revised, constitute the best and most uniform system of parallaxes with which to compare trigonometric parallaxes. Spectroscopic parallaxes have one notable advantage. They depend on purely physical considerations, and whatever errors they may have should be entirely independent of the star's right ascension and declination. It must be borne in mind, however, that the published parallaxes for stars of different spectral types depend on different calibration-curves, and often, indeed, on different spectral characteristics. It is only for the parallaxes derived from any one of these curves that the independence of position in the heavens may safely be postulated. The calibration-curves depend not on any general theory but on observed trigonometric parallaxes supplemented by information from proper motions and radial velocities. It is possible that any particular curve may therefore be affected by a systematic error either in zero point or in scale. Such errors are presumably small, but so also are the systematic errors of the trigonometric parallaxes at present under discussion. If the proportion of stars of different spectral types varies from one right ascension to another, the average of the spectroscopic parallaxes in the corresponding list may have a systematic error varying with the right ascension.

Besides these errors of calibration, spectroscopic parallaxes are subject to errors of two other types: first, an accidental error arising from uncertainties of the estimated line intensities; second, a real

error due to deviations between the physical conditions in the individual and the conditions for the average star on which the calibration is based. As pointed out by Russell, Dugan, and Stewart,[1] Arcturus is a conspicuous example where the spectroscopic parallaxes are consistently larger than the trigonometric parallaxes. For the individual star, this is a true systematic error in the spectroscopic parallax. At the present time there is not enough information to decide whether these systematic errors will persist in late-type stars or whether they will average out in a large number of stars.

The combined error in the spectroscopic parallax is proportional to the size of the parallax itself. All spectroscopic investigators agree in finding the error to be almost 20 per cent of the parallax. It is well recognized that on this account large spectroscopic parallaxes cannot compete in accuracy with trigonometric values for the same stars. For the general run of naked-eye stars with small proper motion, the trigonometric and spectroscopic determinations are comparable as to probable error. For very small parallaxes, e.g., for the average ninth-magnitude star, the spectroscopic parallaxes would have a corresponding advantage, provided spectra of adequate dispersion are available, provided absorption of light in space does not disturb results, and provided the apparent magnitudes are well known.

For Allegheny and McCormick the differences between trigonometric and spectroscopic parallaxes ($\pi_t - \pi_s$) examined by van Maanen far outnumber the material for the other observatories. The three curves at the top of Figure 1 for the McCormick parallaxes are the Schlesinger (1926) values for systematic errors, $\pi_t - \pi_s$, and similar values from the van Maanen 1928 and 1933 discussions. At the time of Schlesinger's investigation, there were roughly 450 McCormick parallaxes in common with the spectroscopic, while 600 and 720, respectively, were available to van Maanen in his two discussions. In comparing the 1926 and the 1928 curves, it might be assumed that the 150 additional parallaxes available to van Maanen in 1928 had made great changes in the McCormick systematic errors. The differences between the curves are so great that we seem forced

[1] *Astronomy*, p. 875.

to conclude that possibly neither curve represents the McCormick system of parallaxes.

The difference between the 1928 and 1933 van Maanen curves is essentially one of zero point. Again this difference might be ex-

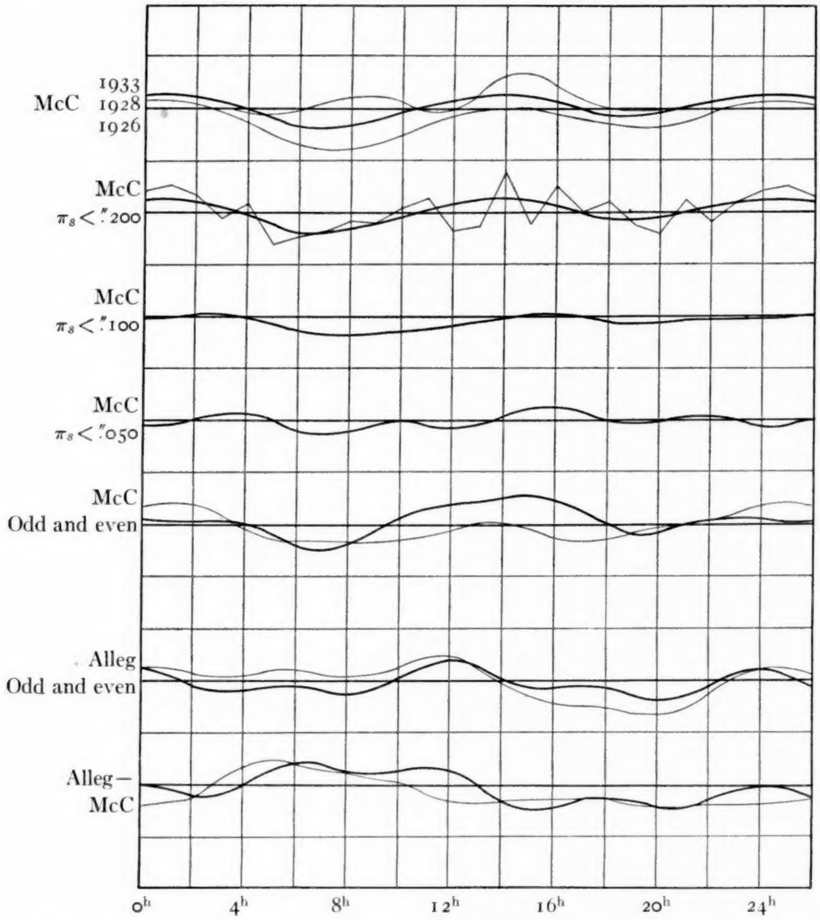

FIG. 1.—Mean differences $\pi_t - \pi_s$ grouped according to right ascension. Scale of ordinates: 1 square $= 0''.01$. Hourly means are plotted for curve second from top.

plained as a change in the McCormick system. An equally good explanation, however, is a change in zero point for the Mount Wilson spectroscopic parallaxes. The Schlesinger 1926 curve for McCormick agrees fairly well in zero point with the van Maanen 1933 curve, but

there is little agreement between the details of the two curves, the humps of the two occurring at different hours of right ascension. As stated above, by following the same methods (except in details) the three McCormick curves purport to do the same thing, namely, give the systematic errors of the McCormick parallaxes after the relative parallaxes by the trigonometric method have been changed to absolute. Apparently the cause for the wide differences cannot be found exclusively in the McCormick parallaxes. Before pursuing the subject farther, it will be well to have an idea of the McCormick observing program, its resemblances, and its differences from other trigonometric programs, especially Allegheny.

In the reports of the Commission on Parallaxes and Proper Motions of the International Astronomical Union, it is found that the Allegheny program consists essentially of the bright stars down to magnitude 5.5, with comparatively few stars of B and O types. The backbone of the McCormick program has been the same as that of the Allegheny program, with the exception that McCormick has not fought shy of early-type stars. Up to date, nearly one hundred B-type stars have been measured. In addition, the McCormick program has been very rich in proper-motion stars, double stars, stars of large radial velocity, and stars of special interest.

With almost equal numbers of stars for Allegheny (750) and McCormick (737) found in van Maanen's 1933 discussion, the numbers of stars of the two programs, within certain limits of the size of the spectroscopic parallaxes, π_s, are given in Table I. The table shows the relatively greater richness of the McCormick program in stars with larger parallaxes, most of these being dwarfs. This has important consequences when differences are taken between the trigonometric and spectroscopic parallaxes. It is generally recognized by all observers, spectroscopic as well as trigonometric, that when the parallaxes are greater than $0\overset{''}{.}050$, the trigonometric values (expressed in seconds of arc) have a greater accuracy than the spectroscopic, one cause of the uncertainty of the latter parallaxes being the unreliability of the apparent magnitudes of the fainter proper-motion stars.

Another interesting comparison between the Allegheny and the McCormick parallaxes are the differences Allegheny − McCormick,

arranged according to spectral type. In Table II are the differences derived from the van Maanen material after the reduction of the relative parallaxes to absolute, and after making proper allowance for the fact that the Allegheny comparison stars on the average are

TABLE I

COMPARISON OF ALLEGHENY AND McCORMICK PROGRAMS

π_s	Allegheny No.	McCormick No.	π_s	Allegheny No.	Allegheny Per Cent	McCormick No.	McCormick Per Cent
>0".200	5	16	>0".200	5	0.7	16	2.2
Between 0".100 and 0".200	20	39	> .100	25	3.4	55	7.5
Between 0".050 and 0".100	90	136	> .050	115	15.3	191	25.9
<0".050	635	546	< .050	635	80.6	546	64.4
			Total	750	......	737	......

TABLE II

DIFFERENCES,* ALLEGHENY−McCORMICK, ARRANGED ACCORDING TO SPECTRAL TYPE

Spectral Type	Allegheny—McCormick	n
A and B	−3.0	80
F	−2.1	101
G	−1.8	120
K	−1.3	123
M	+1.0	31
Mean	−1.8	455

* Unit = 0".001.

fainter than are used at McCormick. In Table II are listed 455 stars, 13 more than were given by van Maanen. This is the total number of stars common to Allegheny and McCormick which have also spectroscopic parallaxes.

These values show (1) that there are systematic differences between Allegheny and McCormick and (2) that there is a progressive change with spectral type. The simplest explanation of the second

effect is atmospheric dispersion. In an important article Schlesinger[2] shows that for a photographic telescope the amount of atmospheric dispersion introduced into a photograph taken ten minutes of time from the meridian is $-0\overset{''}{.}003$ for a star of B type and $+0\overset{''}{.}003$ for stars of K type. For hour angles of twenty minutes Davidson and Greaves[3] find that atmospheric dispersion will affect the parallaxes by $-0\overset{''}{.}009$ for stars of B type and $+0\overset{''}{.}005$ for M-type stars. Schlesinger[2] makes the following remark:

> It is a well known fact, borne out by nearly every parallax series for which the data have been published, that there is a tendency to make the evening observations at larger hour angles (farther west) than the morning observations. Even so small an average difference in this direction as 5 minutes of time may introduce a systematic error of $0\overset{''}{.}001$ for certain classes of spectra, a quantity that has been shown to be of importance in some of the more recent and accurate series of observations of this class.

This hour-angle effect, introduced by atmospheric dispersion into trigonometric parallaxes by following the usual practice of trigonometric observers, will make the parallaxes of stars of B and A types too small, while those for K and M types will be too large. These effects of atmospheric dispersion are for photographic telescopes, reflectors and refractors. For visual telescopes which photograph with yellow light, the effects are negligible.

Several years ago, following the suggestion of Schlesinger that "trigonometric parallaxes of stars of B type are persistently negative," I investigated the parallaxes of more than sixty stars of this type derived at the McCormick Observatory and found only the proper number of negative parallaxes that could be expected from the theory of errors. No correlation was found to exist between the parallaxes and hour-angle differences of evening and morning plates.

Many years ago Hertzsprung[4] called attention to the effects of atmospheric dispersion in photographs of double stars. He found that with a photographic telescope a star of A type at a zenith distance of 45° has its position elevated by refraction by the considerable angle of $0\overset{''}{.}18$ more than for a star of K type. At the same time he showed that with a visual telescope and a yellow filter the effects of atmospheric dispersion are negligible. A simple test is made by

[2] *A.J.*, **36**, 169, 1926. [3] *M.N.*, **83**, 56, 1922. [4] *A.N.*, **192**, 309, 1912.

photographing a double star having strong contrasts in color, like β Cygni.

As is well known to all trigonometric parallax observers, about ten years ago, after the effects of atmospheric dispersion were realized, the methods of taking photographs for parallax determinations with photographic telescopes were changed so that now all photographs are taken within a few minutes of hour angle from the meridian. It is fortunate that photographs with visual telescopes may be made at much larger hour angles without any fear of effects of atmospheric dispersion. Compared with Allegheny, with its photographic refractor, the McCormick program with a visual refractor works under the severe handicap that to attain the same limiting magnitude it is necessary to expose about ten times as long as at Allegheny. In carrying out a big program of parallax work, the McCormick observers must be content with two exposures per plate, while the Allegheny observers have no such limitations, and, moreover, at McCormick work must be continued with parallax factors as small as 0.5 while with the faster refractor at Allegheny parallax plates may be stopped in the evening sky and started in the morning hours with parallax factors not less than 0.7. The combined result is that, with the same number of plates, the Allegheny observers have more photographic images to measure and the plates have larger average parallax factors. Consequently, the average probable error of the Allegheny parallaxes is $\pm 0''.008$, while at McCormick the average value is $\pm 0''.010$.

The large numbers of stars common to Allegheny and McCormick provide an excellent test for the theory of errors. It is my experience that astronomers who are not parallax observers have rather hazy ideas of what is actually meant in practice by probable error as applied to parallaxes. Many seem to think that when a parallax is measured as $+0''.030 \pm 0''.010$, the parallax must be somewhere between the limits $+0''.020$ and $+0''.040$. If this were so, no two independent parallax determinations could differ by more than twice the probable error. Some astronomers who should know better point with scorn at two parallaxes which differ three or more times their probable errors. As a matter of fact, the largest single difference between Allegheny and McCormick is for the star 28 Cassiopeiae, where

the Allegheny value (reduced to absolute) is $+0''.071$ and McCormick $0''.000$, or the two values differ by seven times the probable error of either. The trigonometric parallax observers are much concerned when such discordances are found—but usually there is nothing to do about it.

The probable error of $\pm 0''.008$ for Allegheny and $\pm 0''.010$ for McCormick represents the internal agreement. The probable error of the difference is $\pm 0''.013$. The left half of Table III shows a comparison of the number of differences found by theory and by observation. There is a fair degree of agreement between theory and practice, except that more large differences are found than are expected. Theory shows that in 20 cases out of 455, Allegheny and McCormick may be expected to have differences between them of $0''.040$ and greater, or more than three times the probable error of a single difference. Instead of 20 stars by theory, 31 are actually found by practice. This excess of large differences led Gauss many years ago to introduce into the theory[5] the "probable error of the probable error." The average difference, Allegheny − McCormick, without regard to sign, is $0''.0165$, which multiplied by 0.8453 gives $\pm 0''.014$ as the probable error of one difference, corresponding to a probable error of Allegheny of $\pm 0''.009$ and of McCormick $\pm 0''.011$. In the right-hand column of Table III is given the number of differences

TABLE III

THE THEORY OF ERRORS: COMPARISONS FOR ALLEGHENY AND McCORMICK

Probable Errors	Theory	Observation ($0''.013$)	Observation ($0''.014$)
$\lesseqgtr$ Probable error	228	236	245
Between 1.0 and $1\frac{1}{4}$ P.E.	45	31	44
Between $1\frac{1}{4}$ and $1\frac{1}{2}$ P.E.	40	50	34
Between $1\frac{1}{2}$ and 2 P.E.	61	51	61
Between 2 and $2\frac{1}{2}$ P.E.	39	33	24
Between $2\frac{1}{2}$ and 3 P.E.	22	23	21
>3 P.E.	20	31	26
Total	455	455	455

[5] See Schlesinger, *op. cit.*, **38**, 189, 1928.

found by observation when the probable error of one difference is taken as $\pm 0\overset{''}{.}014$.

The fourth column shows a better agreement with theory than is found in the third column. In the last line more large differences are found by observation (26) than are demanded by theory (20). However, for differences more than twice the probable error, theory requires 81 cases, the internal probable error ($0\overset{''}{.}013$) shows 87 cases, while in the right column there are only 71 of these large differences. The first line of the table shows more small differences (i.e., less than the probable error) found in practice than is required by theory. The

TABLE IV

MEAN DIFFERENCES, $\pi_t - \pi_s$, FOR DIFFERENT SPECTRAL TYPES

Types	All Observatories		Allegheny and McCormick	
A	$-0\overset{''}{.}0021$	162	$-0\overset{''}{.}0065$	97
F	$+\ .0005$	661	$-\ .0023$	385
G	$-\ .0002$	769	$-\ .0007$	436
K	$+\ .0027$	697	$+\ .0014$	408
M	$+0.0015$	315	$+0.0016$	140
Total	$+0.0009$	2604	-0.0007	1466

usual result of similar comparisons is to find an excess of large errors and also an excess of small ones.

The differences, $\pi_t - \pi_s$, tabulated by van Maanen for stars where $\pi_s < 0\overset{''}{.}200$ seem to show a systematic trend with spectral type. Table IV gives the mean differences for all observatories together and also for the spectroscopic parallaxes compared with the weighted mean of Allegheny and McCormick.

Allegheny and McCormick together have measured 56 per cent of all trigonometric parallaxes. The differences in Table IV show that the spectroscopic parallaxes agree with the trigonometric parallaxes in satisfactory fashion in zero point, but there are systematic differences, the spectroscopic parallaxes giving larger values than the trigonometric for early-type stars and smaller values for late-type stars. It should be recognized that the spectroscopic parallaxes of A-type stars are necessarily much inferior in precision to those for stars of later types. Attention also should be called to the fact

that the spectroscopic parallaxes used in these comparisons are not the final Mount Wilson values, but temporary ones from the reductions of Adams. The spectroscopic parallaxes are now in process of adjustment[6] so as to get rid of the systematic differences depending on spectral type.

Van Maanen's 1933 discussion shows a much closer agreement between trigonometric and spectroscopic parallaxes than was found at the time of his 1928 reduction. It has been contended by van Rhijn[7] that the earlier Mount Wilson spectroscopic parallaxes had systematic errors depending both on spectral type and on absolute

TABLE V

DIFFERENCES,* $\pi_t - \pi_s$, FROM VAN MAANEN'S 1928 DISCUSSION

Types	All Observatories		Allegheny		McCormick	
All types	−22	1650	−26	496	−27	593
Late types	12	950	27	312	13	313
A type	37	210	56	80	43	80
B type	52	63	31	7	46	45
A and B types	41	273	54	87	44	125
Dwarfs	−41	405	−80	84	−57	163

* Unit = 0.″0001.

magnitudes. Although I cannot subscribe to van Rhijn's ideas of zero point, the values $\pi_t - \pi_s$ from my own (unpublished) compilation derived from van Maanen's 1928 discussion are given for Allegheny, McCormick, and all observatories combined.

The persistence of negative signs in the table shows an error in zero point in the spectroscopic parallaxes. The cause is now well known, namely, the use of a correction +0.″005 to reduce trigonometric relative parallaxes to absolute, a reduction too large, on the average, by about 0.″0015. An effect of this zero-point correction is found in Figure 1 in the systematic difference between the 1928 and 1933 curves for the McCormick parallaxes. As is also well known, Adams and his co-workers some time ago recognized that the spectroscopic parallaxes for A and B stars in Table V were too large in value, and had a smaller degree of reliability than was found for

[6] This revision has been completed by the three observers Adams, Joy, and Humason.

[7] *M.N.*, **82**, 744, 1932.

late-type stars. In the spectroscopic parallaxes soon to be published, the Mount Wilson observers have derived new methods for obtaining values for early-type stars.

The average textbook on astronomy of recent date generally gives the impression that spectroscopic parallaxes have a higher accuracy than trigonometric. The better textbooks qualify their statements. In comparing the McCormick parallaxes with the Mount Wilson spectroscopic, one is not surprised to find a very large difference, $\pi_t - \pi_s$, for Barnard's star, amounting to $+0\overset{''}{.}169$. There are seven trigonometric parallaxes of this interesting star of large proper motion, and compared with the mean of the trigonometric parallaxes, $\pi_t - \pi_s$, is $+0.165$. On account of the inferior accuracy of the spectroscopic parallaxes of nearby stars, van Maanen made a wise decision in omitting from his discussion of systematic errors all stars with spectroscopic parallaxes greater than $0\overset{''}{.}200$.

In comparing the 721 McCormick parallaxes where $\pi_s < 0\overset{''}{.}200$ with the spectroscopic, it is found that the average difference, $\pi_t - \pi_s$, without regard to sign is $0\overset{''}{.}0175$, which is larger than the difference, $0\overset{''}{.}0165$ between McCormick and Allegheny. The reason for the larger average difference, $\pi_t - \pi_s$, for McCormick is found in a comparatively few large discordances—much larger, in fact, than take place in comparisons with Allegheny. Needless to say, the stars concerned are dwarfs where the spectroscopic parallaxes have an inferior accuracy. For Krueger 60, for which the trigonometric parallax is well determined, but for which the spectroscopic parallax is less than $0\overset{''}{.}200$, the difference $\pi_t - \pi_s$ for McCormick is equal to $+0\overset{''}{.}112$. In 14^h of right ascension there are three large differences (the names of the stars are not given on account of the temporary character of the spectroscopic parallaxes) of $+0\overset{''}{.}079$, $+0\overset{''}{.}094$, and $+0\overset{''}{.}059$. Fig. 1 shows high hourly point.

In the third column of Table VI are given the average differences, $\pi_t - \pi_s$, without regard to sign, for the McCormick stars found in Table I. The probable error of one difference, $\pi_t - \pi_s$, is obtained by multiplying the values in the third column by 0.8453. Assuming for McCormick a probable error of $\pm 0\overset{''}{.}011$ (as found from the comparisons with Allegheny), the probable errors of the spectroscopic parallaxes are derived and are given in the fifth column. As is to be ex-

pected, these are about 20 per cent of the size of the spectroscopic parallax.

Neglecting all stars where $\pi_s > 0''.100$, there are 682 McCormick stars in common to the spectroscopic parallaxes, a number more than 50 per cent greater than the number of stars common to Allegheny and McCormick. Still further omitting all stars where

TABLE VI

PROBABLE ERRORS OF SPECTROSCOPIC PARALLAXES FROM MCCORMICK STARS

π_s	No. of Stars	Aver. Diff. $\pi_t - \pi_s$	P.E. $\pi_t - \pi_s$	P.E. π_s
$>0''.200$	16	$\pm 0''.054$	$\pm 0''.046$	$\pm 0''.045$
Between $0''.100$ and $0''.200$	39	.038	.032	.030
Between 0.050 and 0.100	136	.021	.018	.014
<0.050	546	.015	.013	.007
<0.200	721	± 0.0175	± 0.015	± 0.010

TABLE VII

THE THEORY OF ERRORS: COMPARISONS OF MCCORMICK PARALLAXES WITH SPECTROSCOPIC PARALLAXES

Probable Error	Theory	Observation
$\leqq$ Probable error	273	284
Between 1.0 and $1\frac{1}{4}$ P.E.	55	39
Between $1\frac{1}{4}$ and $1\frac{1}{2}$ P.E.	48	61
Between $1\frac{1}{2}$ and 2 P.E.	73	73
Between 2 and $2\frac{1}{2}$ P.E.	47	45
Between $2\frac{1}{2}$ and 3 P.E.	27	19
>3 times P.E.	23	25
Total	546	546

$\pi_s > 0''.050$, the number of McCormick stars is the considerable total of 546. Since the probable error of one difference is $\pm 0''.013$, Table VII gives the comparison of theory and practice.

A probable error of one difference amounting to $\pm 0''.013$ may be considered as made up of $\pm 0''.010$ coming from the internal agreement of McCormick and $\pm 0''.008$ for Mount Wilson spectroscopic, or $\pm 0''.011$ for McCormick and $\pm 0''.007$ for spectroscopic parallaxes.

From Table VI it would seem evident that in order to diminish the effects of accidental errors in the comparisons of any given series of trigonometric parallaxes with the spectroscopic, all stars where $\pi_s > 0''.050$ should be omitted from the differences $\pi_t - \pi_s$, so that the final results may show the *systematic* errors of the trigonometric parallaxes freed as much as possible from the effects of a very few large *accidental* errors coming chiefly from the spectroscopic values.

It has repeatedly been urged that the spectroscopic parallaxes are now very numerous and that it is utterly impossible that spectral line intensities, on which absolute magnitudes and spectroscopic parallaxes depend, can have any seasonal effects or be influenced in any manner by the right ascension of the star. In stressing this argument, certain elementary facts are overlooked. The final values derived for the systematic errors of the trigonometric parallaxes for a given observatory depend exclusively on the individual stars involved in the differences $\pi_t - \pi_s$. Instead of four thousand spectroscopic parallaxes as a basis for discussion of systematic errors, there are just as many spectroscopic parallaxes entering into the discussion of one observatory as there are trigonometric parallaxes for that observatory. Take as an extreme case discussed by van Maanen, that of the Cape photographic parallaxes with ninety-two stars, or four stars per hour of right ascension. For so few stars there is no spectroscopic system, especially since many of the stars concerned have large accidental errors in the spectroscopic parallaxes. The probable error of one hourly point derived from the differences $\pi_t - \pi_s$ is $\pm 0''.006$, so that it would be reasonably expected that the *accidental* errors would have a range equal to twice the probable error, or from $-0''.0116$ to $+0''.0069$, the values given by van Maanen as *systematic* errors.

Without pursuing these points farther, let us look more closely into van Maanen's curves for McCormick and Allegheny. I personally can see no particular reason for following the method of taking three-hour means as was done by him in 1928 and again in 1933. Of course, this process smoothes out the observational errors—but that is a minor consideration. In Table VIII is found a tabulation for the seven constants for Allegheny and McCormick, various samples of the curves for McCormick being found in Figure 1. The

constants, derived by least squares, in all cases but one were obtained by following exactly van Maanen's methods of three-hour means. The one exception is for McCormick, marked "Hourly Points," where the individual hourly means were taken instead of the rounded three-hour means. Although different numbers of stars were involved per hour of right ascension, it did not seem worth while to do other than employ equal weights. As is seen in Table

TABLE VIII

FOURIER CONSTANTS FOR ALLEGHENY AND MCCORMICK DIFFERENCES* FROM SPECTROSCOPIC PARALLAXES

CONSTANTS	McCORMICK						ALLEGHENY		
	$\pi_s < 0''.200$				$\pi_s < 0''.100$	$\pi_s < 0''.050$	$\pi_s < 0''.200$		
	3-Hour Means	Hourly Points	Odd	Even			All Stars	Odd	Even
a_0	0.0	0.0±0.5	+0.7	−0.8	−1.1	0.0	−0.6±0.4	−0.8	−0.5
a_1	+0.4	+0.8± .7	−1.3	+2.2	+0.9	+0.2	−0.9± .5	−0.8	−1.0
b_1	+2.1	+2.1± .7	+2.1	+2.1	+c.2	−0.1	+2.4± .5	+2.1	+2.5
c_1	−0.3	−0.6± .7	−0.5	−0.1	−0.3	−0.9	+1.4± .5	+1.5	+1.2
a_2	−1.2	−1.3± .7	−2.1	−0.3	−1.0	−1.0	+2.0± .5	+0.3	+3.7
b_2	+1.4	+1.3± .7	+1.9	+1.c	+1.2	+1.0	+0.4± .5	+0.3	+0.5
c_2	0.0	−0.1±0.7	−1.0	+1.0	−0.3	−0.8	−0.3±0.5	−0.5	−0.2

* Unit = $0''.001$.

VIII, the three-hour means and the hourly points give very nearly the same constants. Constants are given for McCormick (but not for Allegheny) after discarding stars where $\pi_s > 0''.100$ and $\pi_s > 0''.050$, respectively.

The constant, a_0, is the value $\pi_t - \pi_s$ would have if the other six constants were zero or no periodic terms depending on right ascension. In other words, a_0 is the straight mean of all hourly differences between trigonometric and spectroscopic parallaxes. It is interesting to notice that for the 721 McCormick stars, where $\pi_s < 0''.200$, a_0 is $0''.0000$. Omitting the very small number of 39 dwarfs where $\pi_s > 0''.100$, then 682 McCormick stars have a different zero point, $-0''.0011$. The reason is that the 39 stars, mainly of K and M types, have an accumulated difference $\pi_t - \pi_s = +0''.634$. Again with 546 McCormick stars where $\pi_s < 0''.050$, a_0 has changed back to $0''.0000$.

Attention should be called to the fact that the McCormick system has not changed in any way in these three different comparisons with the spectroscopic parallaxes.

The third curve from the top of Figure 1 represents McCormick where $\pi_s < 0''.100$. The pronounced bump above the horizontal line near 14^h of right ascension, found in the curve immediately above, has practically disappeared. The fourth curve from the top is McCormick where $\pi_s < 0''.050$.

The "odd" and "even" constants and curves (found fifth from the top in Fig. 1) were obtained by exactly the same methods followed with van Maanen's 1928 discussion. I took the 721 McCormick parallaxes and divided them into two series, the first, third, fifth, etc., parallaxes in order of right ascension going into the odd group, the remainder into the even group. In order to be impartial, I took no part in dividing the stars into the two groups. This simple selection was done by Dr. D. Reuyl, who performed all of the reductions by least squares. Hence there are 361 McCormick stars in the odd group, and 360 in the even. A similar division was made for Allegheny. Either group, odd or even, of Allegheny or McCormick contains 50 per cent more stars than the total of any other single observatory. The constants found in Table VIII, or the curves in Figure 1, *show as great or greater differences between the McComick odd and the McCormick even than are found to exist between Allegheny and McCormick.*

For the 721 McCormick parallaxes where $\pi_s < 0''.200$, the probable errors of the Fourier constants are $\pm 0''.0005$ for a_0 and $\pm 0''.0007$ for the other six constants. By division into odd and even, the number of stars in each group is divided by 2 and hence the probable errors are $\pm 0''.0007$ for a_0 and $\pm 0''.0010$ for the other constants. By omitting 39 stars with comparatively large accidental differences, the constants and curve from 682 stars where $\pi_s < 0''.100$ are the result of fewer stars, with smaller accidental errors, than come from the total of 721 stars, and hence the probable errors of the harmonic constants for the 682 stars are of about the same size as for the 721 stars. Similarly, the probable errors for 546 stars, where still more stars with large accidental errors are omitted, are found to be $\pm 0''.0005$ for a_0 and $\pm 0''.0007$ for the other six constants.

For the 721 McCormick stars used by van Maanen, where $\pi_s < 0''.200$, it is found that for the seven Fourier constants, four are smaller than the probable error, r, two have values lying between r and $2r$, and only one constant, b_1, is greater than twice the probable error. For the 682 and 546 McCormick stars, respectively, none of the constants is as large as twice the probable error. The only large constant, b_1, found for 721 stars, has practically disappeared in the solutions for 682 and 546 stars, respectively. Hence it would seem that the Fourier constants found for each of the three separate McCormick solutions for 721, 682, and 546 stars are exactly what one might expect on the assumption that the differences $\pi_t - \pi_s$ are entirely the result of *accidental* errors. In other words, the curve found by van Maanen for the McCormick parallaxes gives no information regarding *systematic* errors. Similar conclusions cannot be drawn for the Allegheny parallaxes for the reason that two of the seven constants are smaller than the probable error, r, two lie between r and $2r$, and three are greater than $2r$.

The differences between the McCormick odd and even are greater than are found from similar comparisons for Allegheny, thus showing larger accidental errors, partly a result of the greater richness of the McCormick program in dwarfs, as shown in Table 1.

Comparing the Fourier constants for McCormick odd and even, it is seen that the differences between odd and even are of such sizes that three of the seven differences are less than the probable error, three lie between r and $2r$, and one difference, for a_1, exceeds twice the probable error. In the odd and even groups the McCormick system is identical for both, and likewise the spectroscopic system is identical. The differences between the odd and even series show plainly the effects of *accidental* errors. Therefore, if the great differences found between the McCormick odd and even parallaxes are merely the result of accidental errors, we cannot have any confidence in the assumption that the two series added together will entirely eliminate all accidental errors and leave nothing remaining but the systematic errors of McCormick.

The reliability of the "systematic" corrections of van Maanen for Allegheny and McCormick may be tested in another manner. Coefficients derived by him for Allegheny and McCormick, and

given in Table VIII, are repeated in Table IX. Likewise there is given the difference of the coefficients (Allegheny – McCormick) and also the coefficients found by van Maanen[8] from the direct differences of the two series of trigonometric parallaxes. The differences between the values in the fourth and fifth columns are given in the sixth column.

If the differences are taken between the corrections for Allegheny and McCormick as found in *Mount Wilson Contribution* No. 474, page 479, and are plotted for each hour of right ascension, we have

TABLE IX*

COMPARISONS OF ALLEGHENY AND MCCORMICK PARALLAXES

(1) Constants	(2) Alleg. $\pi_t - \pi_s$	(3) McC. $\pi_t - \pi_s$	(4) Alleg. – McC. $\pi_t - \pi_s$	(5) Alleg. – McC. π_t	(6) Difference (4) and (5)
a_0	−0.6	0.0	−0.6	−1.4±0.7	0.8
a_1	−0.9	+0.4	−1.3	−0.5±0.9	0.8
b_1	+2.4	+2.1	+0.3	−2.2	2.5
c_1	+1.4	−0.3	+1.7	+0.1	1.6
a_2	+2.0	−1.2	+3.2	+3.6	0.4
b_2	+0.4	+1.4	−1.0	+0.1	1.1
c_2	−0.3	0.0	−0.3	−0.9±0.9	0.6

* Unit = 0.″001.

the results given at the bottom of Figure 1. By the interpretation of van Maanen, the two curves ought to represent the same physical fact, namely, the systematic difference between Allegheny and McCormick, one of the curves being derived from straight differences, Allegheny – McCormick, the other from similar differences resulting from comparisons with spectroscopic parallaxes. The two curves differ considerably, particularly around 12^h of right ascension, where as a matter of fact the stars common to the two observatories are fewer than average. Of course, it should be remembered that both Allegheny and McCormick have more stars in common with the spectroscopic parallaxes than are common to each other, so the material discussed is not identical. The question might reasonably be asked which of the two curves represents more nearly the systematic differences between Allegheny and McCormick. Are the pronounced differences at certain parts of the two curves the result of *systematic*

[8] *Loc. cit.*

or of *accidental* errors? If the result of the latter, possibly owing to the smaller number of stars common to Allegheny and McCormick than are common to spectroscopic, then it should be remembered that van Maanen had 442 stars to form his curve of straight differences, Allegheny−McCormick. If the differences between the two curves are explainable as the result of the particular groups of stars used in certain hours of right ascension, then it is evident that it is necessary to revise our ideas concerning the meaning of *systematic* errors.

Some light perhaps may be thrown on this perplexing question by utilizing van Maanen's detailed results. His discussion of systematic errors published in 1928 dealt with parallax measures made by Allegheny and McCormick over a period of slightly more than ten years. In his second discussion in 1933 measures for an extra five-year period were added. In Table X there are given for McCormick, in the second and fourth columns, the systematic corrections $(\pi_s - \pi_t)$ published by van Maanen[9] in the years 1928 and 1933, respectively. For each of the two columns there is given, immediately below, the algebraic sum of the hourly corrections called "zero point." Applying to the 1928 published values this correction from zero point, the 1928 values thus corrected and those for 1933 should be strictly comparable. The fifth column gives the differences between the corrections for each hour found in the third and fourth columns. A similar treatment is given for Allegheny, except that one column of figures is omitted to simplify printing. The corrections for zero point from the 1928 discussion, $+0\rlap{.}''0026$ for McCormick and $+0\rlap{.}''0027$ for Allegheny, as already stated, come largely from the error in zero point of the spectroscopic parallaxes.

Hence, after reducing the 1928 corrections to the same zero point as the 1933 corrections, the columns headed "1928 Corr." should represent the systematic corrections for a ten-year period, the column of figures "1933–1928 Corr." in the same manner should represent the systematic corrections of the following five-year period, while the algebraic sum of the two represents the systematic corrections in 1933 for the fifteen-year period. The values in the column headed "1933–1928" are the result of any changes in the spectro-

[9] *Ibid.*

scopic system and also any changes in the McCormick (or Allegheny) system. It is evident that if McCormick has a constant "system," the high spots from the 1928 discussion should exhibit themselves

TABLE X

COMPARISONS OF THE 1928 AND 1933 SYSTEMATIC CORRECTIONS OF VAN MAANEN

	McCormick				Allegheny			Allegheny − McCormick		
R.A.	1928	1928 Corr.	1933	1933−1928 Corr.	1928 Corr.	1933	1933−1928 Corr.	1928 Corr.	1933	1933−1928
0^h	−1.3	−3.9	−2.2	+1.7	−0.5	−2.3	−1.8	+3.4	−0.1	−3.5
1	−1.5	−4.1	−2.4	+1.7	−1.4	−1.7	−0.3	+2.7	+0.7	−2.0
2	−0.5	−3.1	−2.2	+0.9	−1.7	−0.1	+1.6	+1.4	+2.1	+0.7
3	+1.0	−1.6	−1.1	+0.5	−1.6	+0.8	+2.4	0.0	+1.9	+1.9
4	+3.1	+0.5	+0.5	0.0	−1.4	+0.6	+2.0	−1.9	+0.1	+2.0
5	+5.3	+2.7	+2.4	−0.3	−1.3	−0.2	+1.1	−4.0	−2.6	+1.4
6	+7.1	+4.5	+3.6	−0.9	−1.5	−0.4	+1.1	−6.0	−4.0	+2.0
7	+8.1	+5.5	+4.0	−1.5	−2.1	+0.4	+2.5	−7.6	−3.6	+4.0
8	+7.9	+5.3	+3.3	−2.0	−2.8	+1.0	+3.8	−8.1	−2.3	+5.8
9	+7.0	+4.4	+2.3	−2.1	−3.8	+0.4	+4.2	−8.2	−1.9	+6.3
10	+5.3	+2.7	+0.8	−1.9	−4.5	−1.7	+2.8	−7.2	−2.5	+4.7
11	+3.3	+0.7	−0.2	−0.9	−4.6	−3.7	+0.9	−5.3	−3.5	+1.8
12	+1.5	−1.1	−1.4	−0.3	−4.1	−4.1	0.0	−3.0	−2.7	+0.3
13	+0.3	−2.3	−2.2	+0.1	−2.8	−2.5	+0.3	−0.5	−0.3	+0.2
14	−0.1	−2.7	−2.8	−0.1	−1.1	+0.3	+1.4	+1.6	+3.1	+1.5
15	+0.2	−2.4	−2.3	+0.1	+1.2	+2.4	+1.2	+3.6	+4.7	+1.1
16	+1.1	−1.5	−1.1	+0.4	+3.4	+3.2	−0.2	+4.9	+4.3	−0.6
17	+2.3	−0.3	+0.2	+0.5	+5.1	+3.2	−1.9	+5.4	+3.0	−2.4
18	+3.1	+0.5	+1.2	+0.7	+6.1	+3.6	−2.5	+5.6	+2.4	−3.1
19	+3.5	+0.9	+1.4	+0.5	+6.3	+4.6	−1.7	+5.4	+3.2	−2.2
20	+3.1	+0.5	+0.9	+0.4	+5.6	+5.2	−0.4	+5.1	+4.3	−0.8
21	+2.2	−0.4	−0.1	+0.3	+4.2	+4.4	+0.2	+4.6	+4.5	−0.1
22	+0.9	−1.7	−1.0	+0.7	+2.5	+1.9	−0.6	+4.2	+2.9	−1.3
23	−0.5	−3.1	−1.6	+1.5	+0.8	−0.9	−1.7	+3.9	+0.7	−3.2
Zero point	+2.6	0.0	0.0	0.0	0.0	+0.6	+0.6	......	......	......
No.	593	593	721	128	496	745	249	......	......	......

as high spots in the five-year period of observations, 1933–1928. The high spot in 1928 comes at 7^h of right ascension, while at the corresponding hour in the column 1933–1928, the large position value is not repeated but there is instead a negative sign. Similarly, the high spot in the 1928 Allegheny values comes at 19^h of right ascension,

while in the five-year period, 1933–1928, the large plus value has been changed to minus.

If differences are taken between the "1928 Corr." values in the sense of Allegheny – McCormick, and likewise for 1933 and for the interval 1933–1928, we have the values given at the right in Table X. Attention should be called to the fact that these differences, Allegheny – McCormick, are not the straight differences but those derived as systematic differences from comparisons with the spectroscopic parallaxes. The column headed "1928" seems to show that unquestionably there were systematic differences between Allegheny and McCormick as the result of the first ten years of parallax measurements at each observatory. To be consistent, the column headed "1933–1928" should likewise be regarded as representing the systematic differences coming from the following five years of measurements. But the trends of the values in the one column have no resemblances whatever to those in the other column. Since Table X gives the means of comparing the systems of parallaxes in the first ten years of work with those of the following five years, the only rational conclusions to draw are that there seems to be no constancy in the Allegheny or McCormick system of trigonometric parallaxes or in the Mount Wilson system of spectroscopic parallaxes.

Lest there be any misunderstanding, I should like to repeat that there is no doubt whatever in my mind that systematic errors have existed between Allegheny and McCormick. Any discussion which refers to all past measures must certainly show that on the average the Allegheny parallaxes are systematically smaller than the McCormick. How to derive the best value of this systematic difference as a function of right ascension, caused possibly by seasonal errors, is a very different and much more perplexing problem.

It is of the utmost importance that the trigonometric parallaxes published by the various observatories shall be reduced by a series of corrections to a common system freed as much as possible from errors of all sources. The importance of the problem is responsible for the many published discussions of systematic errors. It has therefore seemed worth while in the foregoing to compare in detail and from as many different angles as possible the Allegheny and McCormick parallaxes with each other and with the Mount Wilson

spectroscopic values. Without going into similar details with the other contributing observatories, it is readily possible to ascertain from the differences $\pi_t - \pi_s$ furnished me by van Maanen for each observatory what effects the purely accidental errors will have in the formation of the Fourier constants derived by him for each observatory.

From the differences $\pi_t - \pi_s$ for each observatory the average differences without regard to sign are given for each observatory in the first line of Table XI. By multiplying by 0.8453, the probable error, given in the third line, is found for a single difference $\pi_t - \pi_s$. If now the individual trigonometric parallaxes are corrected by van Maanen's systematic corrections,[10] the average differences from the spectroscopic parallaxes taken without regard to sign are given in the second line of the table and the corresponding probable errors in the fourth line. By dividing these probable errors by the square root of the number of stars observed, the probable error given in the fifth line is found for the Fourier constant a_0. The probable errors for six other constants are $\sqrt{2}$ times the probable error of a_0. The values of the seven Fourier constants given in the table are those published by van Maanen.

On the assumption that the errors are entirely accidental, theory states that out of a total of seven harmonic constants, $3\frac{1}{4}$ constants should have values equal to or less than the probable error r, $2\frac{1}{4}$ should lie between r and $2r$, and $1\frac{1}{4}$ might be expected to have values greater than $2r$.

By comparing the values of the average differences given in the first and second lines of the table, or by comparing the probable errors in the third and fourth lines, one is forced to the surprising conclusion that the application of the van Maanen systematic corrections makes practically no changes for four out of the eight observatories, Allegheny, McCormick, Yerkes, and Greenwich. There is a slight improvement made in the results for Mount Wilson, Sproul, and Cape, and a still greater improvement for Johannesburg.

For those who can think in terms of probable errors, the last three lines of the table are instructive. The constant a_0 in the table is the zero-point correction derived by giving equal weights to each hourly

[10] *Ibid.*, Table VII.

TABLE XI

FOURIER COEFFICIENTS AND PROBABLE ERRORS

	M.W. 60	Alleg.	McC.	Yerkes	Sproul	Greenwich	Johannesburg	Cape Ph.
Average difference, $\pi_t - \pi_s$	13.5	12.0	17.5	18.2	18.5	13.8	16.4	17.2
Average difference corrected	12.9	11.7	17.4	18.1	17.3	13.7	14.0	15.9
Probable error	11.4	10.1	14.8	15.4	15.6	11.7	13.9	14.5
Probable error, corrected differences	10.9	9.9	14.7	15.3	14.6	11.6	11.8	13.4
Number of stars	184	745	721	214	192	162	250	92
a_0	$+3.9\pm0.8$	-0.6 ± 0.4	0.0 ± 0.5	$+1.9\pm1.0$	$+4.4\pm1.0$	-0.8 ± 0.9	$+6.7\pm0.7$	$+3.1\pm1.4$
a_1	-2.4 ± 1.2	-0.9 ± 0.5	$+0.4\pm0.7$	-3.9 ± 1.5	-0.9 ± 1.5	-3.5 ± 0.9	-5.8 ± 0.7	-6.7 ± 1.4
b_1	-0.5	$+2.4$	$+2.1$	$+2.4$	$+2.2$	-0.8	$+3.3$	-0.2
c_1	-0.6	$+1.4$	-0.3	-0.4	-0.1	0.0	$+1.3$	-1.4
a_2	$+0.9$	$+2.0$	-1.2	$+1.7$	$+8.1$	-1.7	-4.6	$+0.2$
b_2	$+4.6$	$+0.4$	$+1.4$	-3.4	-2.2	$+0.4$	-2.1	-1.1
c_2	$+1.1\pm1.2$	-0.3 ± 0.5	0.0 ± 0.7	-2.6 ± 1.5	$+0.6\pm1.5$	-3.2 ± 0.9	$+2.1\pm0.7$	$+3.6\pm1.4$
Number of coefficients which are:								
$\lesseqgtr$P.E.	3	2	4	1	3	4	0	4
Between r and $2r$	2	2	2	4	2	1	3	1
$>2r$	2	3	1	2	2	2	4	2

mean. (For Yerkes Observatory, where there are only two, one, and three parallaxes, respectively, in the 10^h, 11^h, and 12^h of right ascension, the zero-point correction from the individual stars is $\pm 0.''0009$ instead of $\pm 0.''0019$ as given in the table.)

The four observatories, Allegheny, McCormick, Yerkes, and Greenwich, where little change is made by the application of systematic corrections, are the observatories where the zero-point corrections are small and of the order of the size of the probable errors. The three observatories, Mount Wilson, Sproul, and Cape, where there are slight improvements by the application of systematic corrections, have larger zero-point corrections, while Johannesburg has the largest correction and the greatest improvement. In spite of the fact that the Yerkes Observatory shows two constants, a_1 and b_2, which in size exceed twice the probable error, the average probable error of one difference is not improved by applying van Maanen's systematic corrections. Similarly, Allegheny has three constants which are greater than twice the probable errors and yet the application of systematic corrections to the individual stars makes little improvement in accuracy.

These conclusions derived from the individual stars are greatly at variance with the conclusions of van Maanen after applying his systematic corrections to three-hour means.

Taking all factors into consideration, one is forced to the conclusion that the six last Fourier constants have little physical significance or little practical bearing on the question of systematic errors. The simplest method, therefore, of deriving a systematic correction for each separate series of trigonometric parallaxes is to find the zero-point correction, preferably from the individual stars and not from the hourly means.

CONCLUSIONS

As has been already stated, the first decade of trigonometric parallax work by the Allegheny, McCormick, and Mount Wilson observatories showed that on the average the McCormick parallaxes were consistently larger than the Allegheny values and smaller than those of Mount Wilson. As far as I know, no one has an adequate explanation for these persistent systematic differences. With the

second decade of parallax measures, the systematic differences between Allegheny and McCormick, as shown by Table X, ceased to exist, or, to express it more scientifically, became inappreciable in size. For reasons which need not be specified, the trigonometric parallaxes now being completed at each and every observatory have a higher degree of reliability than the earlier parallaxes when each observatory was publishing results as quickly as possible. In the present discussion, Allegheny and McCormick with their very large numbers of parallaxes, have been compared with each other and with the Mount Wilson spectroscopic parallaxes. These comparisons show that systematic differences between Allegheny and McCormick unquestionably have existed in the past, but the differences are now much smaller than formerly.

By dividing into "odd" and "even" series the 721 McCormick stars used by van Maanen, one sees at a glance the effects of accidental errors. Hence the underlying fundamental assumption that accidental errors balance out in the long run and have no cumulative effects is not borne out. To diminish the effects of the accidental errors of the spectroscopic parallaxes, dwarf stars with inferior spectroscopic accuracy should be discarded from the comparisons.

For the McCormick parallaxes, each different selection of material gives a different set of constants in Table VIII and different curves in Figure 1. If one wishes for McCormick a system of corrections for each hour of right ascension, unquestionably the curve which approximates the truth most closely will be the fourth from the top in Figure 1, where all stars are discarded where $\pi_s < 0\overset{''}{.}050$. The probable error of one difference, $\pi_t - \pi_s$, for these 546 McCormick stars is $\pm 0\overset{''}{.}013$. With an average of 27 stars per hour, the probable error of one hourly point, as the result of *accidental* errors, is $\pm 0\overset{''}{.}0025$. It seems therefore quite probable that even the fourth curve from the top in Figure 1, with its small amplitudes, shows solely the effects of *accidental* errors and consequently has little to do with *systematic* errors.

From the comparisons of several hundred parallaxes common to the McCormick trigonometric and the Mount Wilson spectroscopic parallaxes, it is evident that the *systematic* differences between the two systems are so exceedingly small that they cannot be definitely

detected even with this very large amount of material. The outstanding differences for the hourly means arise mainly, if not entirely, from residual effects of *accidental* errors.

It is my opinion that even for Allegheny and McCormick with their large numbers of parallaxes, all that we know at the present time about systematic errors as a function of right ascension caused by possible seasonal effects is that these errors are very small. To my way of thinking, the only practical procedure for ascertaining the systematic errors of trigonometric parallaxes is to neglect possible seasonal effects and then, by grouping together all stars in all hours of right ascension, find the value of the correction $\pi_s - \pi_t$ for each observatory. This should then be used as the best approximation for the average value of the systematic correction for each observatory from the beginning of parallax work up to the present.

I am greatly indebted to Dr. Adams and Dr. Russell for the opportunity of discussing with them the details of this article.

With these words, I hand over the question to my good friend, Dr. Frank Schlesinger, who with many additional trigonometric parallaxes and the new Mount Wilson system of spectroscopic parallaxes will discuss anew in the forthcoming *Yale Catalogue* the perennial problem of systematic errors of parallaxes.

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June 1934

NOTES

RELATIVE STELLAR ENERGY DISTRIBUTION IN THE INFRA-RED

ABSTRACT

The relative spectral intensities of five stars have been determined at 0.87 μ and 1.0 μ by the use of the new Eastman special infra-red emulsions. The results extend the wave-length region observed by Jensen and confirm his conclusion that differential stellar energy-curves deviate markedly from the approximately linear relationship to be expected from Planck's law.

It has been known for some time that the spectral energy distribution of late-type stars deviates systematically in the violet from black-body distribution, but it is only recently that definite evidence has been found of a divergence between observed and theoretical curves for stars of all spectral types throughout the range $\lambda\lambda$ 3600–6400. As the differences in magnitude at corresponding spectral regions between two stars of different temperatures should be, according to Planck's law, approximately a linear function of $1/\lambda$, a straight-line relationship has been assumed in color-temperature work.

The lack of agreement in the slope of this line, as determined from different spectral regions, led Jensen[1] to investigate the nature of the observed relationship. His results show that, instead of a linear relationship, a very definite curvature exists and that the problem of determining stellar color temperatures is considerably more complicated than has been realized.

It has become of considerable interest to extend the observed spectral regions as far as possible toward the infra-red, in order to investigate further the behavior of stellar radiation. The energy distribution in the far infra-red has been observed radiometrically for nineteen stars by Abbott. His measures extend to 2.2 μ for most of the objects observed. For a number of these stars the spectral intensity distribution was extended through the ordinary visual and photographic regions by including the spectrophotometric results of Rosenberg and of Wilsing, Scheiner, and Münch. Black-

[1] *A.N.*, **248**, 217, 1933.

body curves were fitted to the observed intensities, and color temperatures were derived. Abbott states in the conclusion to his latest work on the subject that much improvement is needed before the infra-red spectral energy-curves of any of the stars can be regarded as well determined.

Recent developments by the Eastman Kodak Company in sensitizing dyes have made possible the extension of ordinary photo-

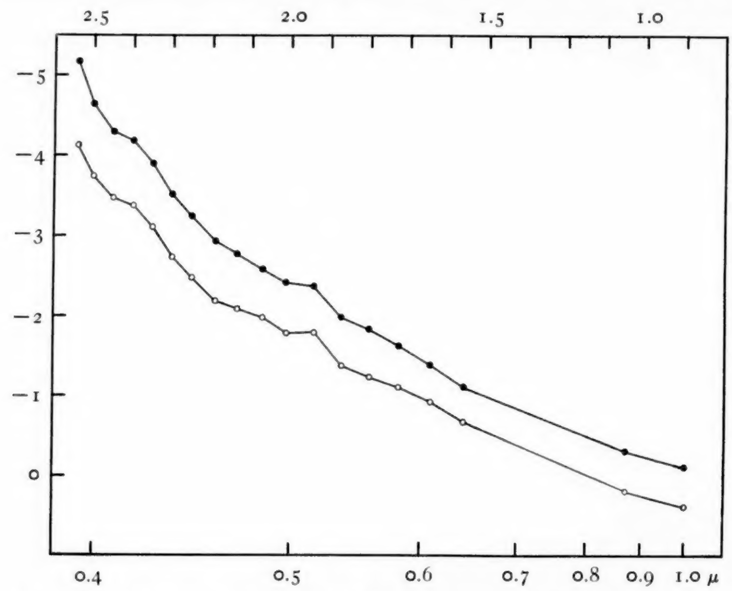

FIG. 1.—Relative spectral intensities for α Lyrae (A0)−β Ursae Minoris (K5) (closed circles) and α Aquilae (A5)−β Ursae Minoris (open circles). Abscissas below, λ expressed in μ; abscissas below, 1/λ; ordinates, difference in magnitude. Data to the violet of λ 6500 are taken from the work of Jensen.

graphic spectrophotometry to much greater wave-lengths than was previously possible. Special emulsions P and Q are sensitive to the regions 0.8–1.2 μ and are of sufficient speed to photograph stellar spectra with short exposures, even with small instruments.

A 15° objective prism crossed by a grating in the method of Hertzsprung was used in connection with a 6-inch reflector of focal ratio 1:10. The plates were calibrated by successive exposures of equal duration on the same star, with and without the grating. The difference in magnitude of the two exposures was found from the grating constants to be 0.7 mag. The stars α Lyrae (A0) and α Aqui-

lae (A5) were compared directly with β Ursae Minoris (K5), while α Aurigae and α Tauri were intercompared. As the difference in intensity at the points 8700 A and 10,000 A was always less than the grating reduction factor of 0.7 mag., the difference in magnitude could be directly estimated with accuracy. It is improbable that errors greater than 0.2 mag. are present. The exposures were 7 minutes in duration for IQ plates and 5 minutes for the IP emulsion. All plates were sensitized in an ammonia bath. The scale of the spectrograms is about 1000 A per mm at λ 9000. The regions

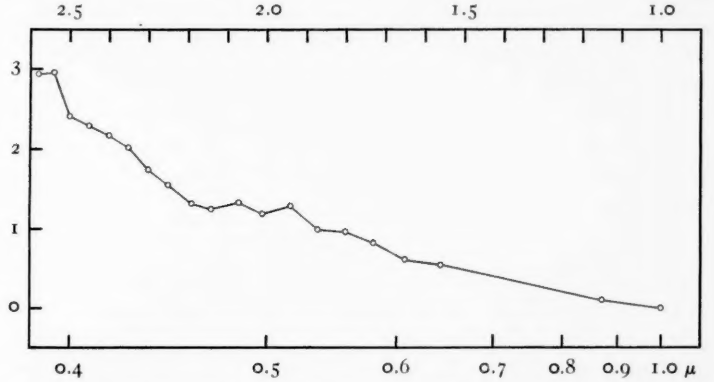

FIG. 2.—Relative spectral intensities for α Aurigae (G0)−α Tauri (K5). The co-ordinates are the same as in Fig. 1. The data to the violet of λ 6500 are taken from the work of Jensen.

included in the intensity estimates are approximately $\lambda\lambda$ 8500–8900 and 9800–10200. These regions are free of any noticeable absorption on the small scale used. The spectra were unwidened.

The data for the region $\lambda\lambda$ 3600–6400 were taken from Jensen, and his curves were extended by means of the additional points in the infra-red. The results are shown graphically in Figures 1 and 2. The curvature found by Jensen is confirmed and is shown to continue as far to the red as it is possible to observe photographically at present. It is seen that it is quite impossible to represent the relative energy-curves linearly, and the method of determining color temperatures by the simple gradient method is open to serious objections.

W. W. MORGAN AND B. A. WOOTEN

YERKES OBSERVATORY
September 26, 1934

REVIEWS

Atomic Theory and the Description of Nature, I: *Four Essays and an Introductory Survey*. By N. Bohr. Cambridge: University Press; New York: Macmillan Co., 1934. Pp. 119. $2.00.

The present volume is a collection of four lectures delivered during the years 1925–1929. All have been published in journals of various languages, and in that form have exerted an important influence on the development of atomic theory during those years. A German edition of the same essays appeared in 1931; the publication of an English edition is an event which has long been desired by all who are interested in the diffusion of knowledge concerning the recent development of physics.

Characteristically, the book contains almost no mathematics and yet is an example of the most rigorous logic. It is to be recommended to all who are interested in the foundations of science. As indicated by the title, the essays are concerned with "the fundamental principles underlying the description of nature," atomic physics being considered as a single field for their application. The phrase is a fortunate one. Modern science is recognizing its twofold purpose: discovery and description. Classic science confused itself by aiming at the discovery of the description of nature, considering the description to be somehow a part of objective nature and in no way limited by our subjective forms of perception. The recognition of the limitations of various forms of perception and their complementary relationships constitutes the greatest of Bohr's contributions to science. It is this which is the subject matter of the essays.

Carl Eckart